WIND ENERGY CONVERSION THEORY, BETZ EQUATION

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2/23/2021

INTRODUCTION

Wind machines performance is described by Betz's theory which applies to horizontal axis wind machines. However, the efficiency of vertical axis wind machines is also estimated relative to the maximum power calculated from Betz's formula.

This fundamental formula was first introduced by the German engineer Albert Betz in 1919 and published in his book: "Wind Energie und ihre Ausnutzung durch Windmühlen," or "Wind Energy and its Extraction through Windmills," published in 1926. The developed theory thus applies to both horizontal and vertical axis wind machines.

Wind-Energie

und ihre Ausnutzung durch Windmühlen

Dipl.-Ing. Dr. Albert Betz

Figure 1. "Wind Energie und ihre Ausnutzug durch Windmülen," "Wind Energy and its Extraction through Windmills," book authored by German physicist Albert Betz, 1926.

FLOW SYSTEM ASSYMETRIES AND PRINCIPLES OF ENERGY CONVERSION

A first basic principle of energy conversion or extraction from the environment can be simply enunciated as:

"Energy can only be extracted from a flow system."

A corollary is that:

"The energy flow is from a high energy storage reservoir to a low energy sink."

In hydroelectric power generation, the potential energy of water blocked behind a dam, a waterfall or a dam on a river or stream cannot be extracted unless it is allowed to flow. In this case only a part of it can be extracted by a water turbine.

In a heat engine, the heat energy cannot be extracted from a totally insulated reservoir. Only when it is allowed to flow from a high temperature reservoir at which heat is added, to a low temperature reservoir where it is rejected to the environment, can a fraction of this energy be extracted by a heat engine.

In geothermal energy production the differential temperature deep underground close to the magma and the Earth surface allows for an energy flow producing steam to drive a steam turbine.

In Ocean Thermal Energy production (OTEC) the cooler temperature deep in the ocean compared with the warmer temperature at the surface allows for the boiling of a low boiling-point working substance such as ammonia (NH3) at the surface and then its condensation at the cooler depth resulting in a flow system. Ocean tidal power generation depends on the flow of water stored at a period of high tide behind a dam to flow out of storage at a period of low tide. Ocean wave production uses the difference in the kinetic energy content in waves from crest to bottom generated by wind flow on the surface of the water.

Totally blocking a wind stream does not allow efficient energy extraction. Only by allowing the wind stream to flow from a high speed region to a low speed region can energy be extracted by a wind energy converter.

A second principle can be stated as:

"Only asymmetries in a hydraulic, kinetic, thermodynamic or aerodynamic system allow the extraction of a portion of the available energy in the system."

Ingenious devices, conceptualized by ingenious people, take advantage of naturally existing asymmetries. Alternatively, ingenious artificial configurations or situations favoring the creation of these asymmetries are created so as to extract energy from the environment.

A third principle is that:

"The existence of a flow system necessitates that only a fraction of the available energy can be extracted at an efficiency characteristic of the energy extraction process, with the rest returned back to the environment."

In thermodynamics, the ideal heat cycle efficiency is expressed by the Carnot cycle efficiency. In a wind stream, the ideal aerodynamic cycle efficiency is expressed by Betz's efficiency equation.

POWER CONTENT OF A FREE FLOWING WIND STREAM

The power content in a cylindrical column of free unobstructed air moving at a constant speed V is the rate of change in its kinetic energy:

$$P = \frac{dE}{dt} \tag{1}$$

Expressing the kinetic energy as:

$$E = \frac{1}{2} m V^2 \tag{2}$$

Substituting from Eqn. 2 into Eqn. 1, we get:

$$P = \frac{dE}{dt} = \frac{d}{dt}(\frac{1}{2} m V^2)$$

Applying the chain rule of differentiation:

$$\frac{d}{dt}(u.v) = u\frac{dv}{dt} + v\frac{du}{dt}$$

Then:

$$\begin{aligned} P &= \frac{1}{2}(\frac{d}{dt}(mV^2) \\ &= \frac{1}{2}(2mV\frac{dV}{dt} + V^2\frac{dm}{dt}) \end{aligned}$$

For a constant wind speed V,

$$\frac{dV}{dt} = 0$$

And consequently the power can be expressed in terms of its speed V and its mass flow rate as:

$$\begin{aligned} P &= \frac{1}{2}(\frac{d}{dt}(mV^2) \\ &= \frac{1}{2}\dot{m}V^2 \end{aligned} \tag{3}$$

If the cross sectional area of the column of air is S, and its density is ρ, the mass flow rate is:

$$\dot{m} = \rho S V \tag{4}$$

By substituting from Eqn. 4 into Eqn. 3, this yields the power content of the column of air as:

$$P = \frac{1}{2}\rho S V^3 \tag{5}$$

If the diameter of the column of air is D, then:

$$P = \frac{1}{2}\rho \frac{\pi D^2}{4} V^3 \tag{6}$$

The power content of the cylindrical column of air is proportional to the *square* of its diameter D, and more significantly to the *cube* of its speed V.

BETZ'S AND CARNOT LAW

Betz's law is analogous to the Carnot cycle efficiency in thermodynamics suggesting that a heat engine cannot extract all the energy from a given hot reservoir of energy and must reject part of its heat input back to the environment. Whereas the Carnot cycle efficiency can be expressed in terms of the kelvin heat input temperature T_1 and the kelvin heat rejection temperature T_2:

$$\eta_{Carnot} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1},$$

Betz's equation deals with the upstream from the turbine wind speed V_1 and the downstream wind speed V_2.

The loss in efficiency of a heat engine is caused by the heat rejection to the environment. The losses in efficiency for a practical wind turbine are caused by the viscous drag on the blades, the swirl imparted to the air flow by the rotor, and the power losses in the transmission and electrical system.

ASSUMPTIONS

Betz developed the global theory of wind machines at the Göttingen Institute in Germany. The wind rotor is assumed to be an ideal energy converter, meaning that:

1. It does not possess a hub.
2. It possesses an infinite number of rotor blades which do not result in any drag resistance to the wind flowing through them.

In addition, uniformity is assumed over the whole area swept by the rotor, and the speed of the air beyond the rotor is considered to be axial. The ideal wind rotor is taken at rest and is placed in a moving fluid atmosphere.

IDEAL WIND ROTOR

Considering the ideal model shown in Fig. 2, we consider that the cross sectional area swept by the turbine blade is S, with the air cross-section upwind from the rotor designated as S_1, and downwind as S_2.

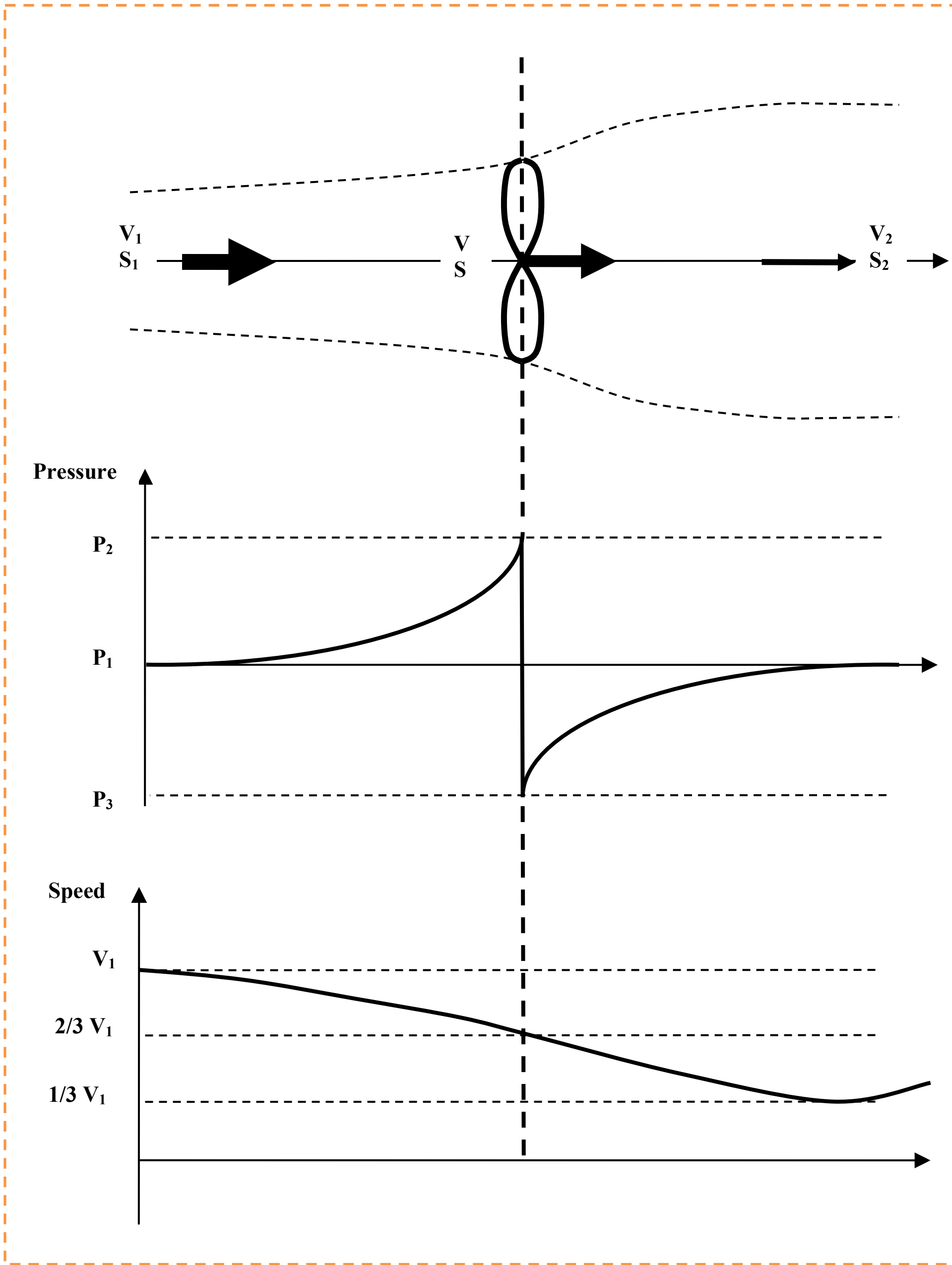


Figure 2. Pressure and speed variation in an ideal model of a wind machine.

The wind speed passing through the turbine rotor is considered uniform as V, with its value as V_1 upwind, and as V_2 downwind at a distance from the rotor.

Extraction of mechanical energy by the rotor occurs by reducing the kinetic energy of the air stream from upwind to downwind, or simply applying a braking action on the wind. This implies that:

$$V_2 < V_1.$$

Consequently the air stream cross sectional area increases from upstream of the turbine to the downstream location, and:

$$S_2 > S_1.$$

The air pressure in front of the turbine increases and sees a sudden drop at the turbine interface, which can be construed to create suction at the other side. Departing from the ideal conditions and increasing that suction by adding a gap at some optimal distance behind the rotor plane and creating a single vortex or a vortex train could in principle enhance the energy extraction process by further reducing the pressure behind the turbine.

CONSERVATION OF MASS, CONTINUITY EQUATION

If the air stream is considered as a case of incompressible flow, the conservation of mass or continuity equation can be written as:

$$\dot{m} = \rho S_1 V_1 = \rho S V = \rho S_2 V_2 = \text{constant} \tag{7}$$

This expresses the fact that the mass flow rate is a constant along the wind stream.

Euler's Theorem gives the force exerted by the wind on the rotor as:

$$\begin{aligned} F &= ma \\ &= m\frac{dV}{dt} \\ &= \dot{m}\Delta V \\ &= \rho SV.(V_1 - V_2) \end{aligned} \tag{8}$$

The incremental energy or the incremental work done in the wind stream is given by:

$$dE = Fdx \tag{9}$$

From which the power content of the wind stream is:

$$P = \frac{dE}{dt} = F\frac{dx}{dt} = FV \tag{10}$$

Substituting for the force F from Eqn. 8, we get for the extractable power from the wind:

$$P = \rho S V^2.(V_1 - V_2) \tag{11}$$

The power as the rate of change in kinetic energy from upstream to downstream is given by applying the law of conservation of energy as:

$$\begin{aligned} P &\simeq \frac{\Delta E}{\Delta t} \\ &\simeq \frac{\frac{1}{2}mV_1^2 - \frac{1}{2}mV_2^2}{\Delta t} \\ &= \frac{1}{2}\dot{m}\left(V_1^2 - V_2^2\right) \end{aligned} \tag{12}$$

Using the continuity equation 7, we can write:

$$P = \frac{1}{2}\rho S V\left(V_1^2 - V_2^2\right) \tag{13}$$

Combining the continuity equation expressing mass conservation to the conservation of energy equation by equating the two expressions for the power P in Eqns. 11 and 13, we get:

$$P = \frac{1}{2}\rho S V\left(V_1^2 - V_2^2\right) = \rho S V^2\left(V_1 - V_2\right)$$

The last expression implies that:

$$\begin{aligned} \frac{1}{2}(V_1^2 - V_2^2) &= \frac{1}{2}(V_1 - V_2)(V_1 + V_2) \\ &= V(V_1 - V_2), \qquad \forall V, S, \rho \neq 0 \end{aligned}$$

or:

$$V = \frac{1}{2}(V_1 + V_2),\ \forall(V_1 - V_2) \neq 0 \ or\ V_1 \neq V_2 \tag{14}$$

This in turn suggests that the wind velocity at the rotor may be taken as the average of the upstream and downstream wind velocities.

It also implies that the turbine must act as a brake, reducing the wind speed from V_1 to V_2, but not totally reducing it to V = 0, at which point the equation is not valid anymore. To extract energy from the wind stream, its flow must be maintained and not totally stopped according to the principles of energy conversion enunciated earlier.

FORCE AND EXTRACTABLE POWER IN TERMS OF THE DOWNSTREAM VELOCITY FACTOR, OR INTERFERENCE FACTOR

The last result allows us to write new expressions for the force F and power P in terms of the upstream and downstream velocities by substituting for the value of V as:

$$\begin{aligned} F &= \rho SV.(V_1 - V_2) \\ &= \frac{1}{2}\rho S.(V_1^2 - V_2^2) \end{aligned} \tag{15}$$

$$\begin{aligned} P &= \rho SV^2\left(V_1 - V_2\right) \\ &= \frac{1}{4}\rho S\left(V_1 + V_2\right)^2\left(V_1 - V_2\right) \\ &= \frac{1}{4}\rho S\left(V_1^2 - V_2^2\right)\left(V_1 + V_2\right) \end{aligned} \tag{16}$$

We can introduce the "interference parameter," "interference factor," or "downstream velocity factor," b as the ratio of the downstream speed V_2 to the upstream speed V_1 as:

$$b = \frac{V_2}{V_1} \tag{17}$$

From Eqn. 15 the force F can be expressed as:

$$F = \frac{1}{2}\rho S.V_1^2(1 - b^2) \tag{18}$$

The extractable power P in terms of the interference factor b can be expressed as:

$$\begin{aligned} P &= \frac{1}{4}\rho S\left(V_1^2 - V_2^2\right)\left(V_1 + V_2\right) \\ &= \frac{1}{4}\rho SV_1^3\left(1 - b^2\right)\left(1 + b\right) \end{aligned} \tag{19}$$

The most important observation pertaining to wind power production is that the extractable power from the wind is proportional to the cube of the upstream wind speed

V_1^3 and is a function of the interference factor b.

POWER FLUX, POWER DENSITY

The "power flux" or rate of energy flow per unit area, sometimes referred to as "power density" is defined using Eqn. 5 as:

$$\begin{aligned} P' &= \frac{P}{S} \\ &= \frac{\frac{1}{2}\rho S V^3}{S} \\ &= \frac{1}{2}\rho V^3, [\frac{Joules}{m^2.s}], [\frac{Watts}{m^2}]. \end{aligned} \tag{20}$$

It is worth noting that he terminology "density" is reserved for volumes (m^3) whereas "flux" is used for unit areas (m^2), and "specific" is used for unit weights (kg).

PERFORMANCE COEFFICIENT C_p, EFFICIENCY

The kinetic power content of the undisturbed upstream wind stream with $V = V_1$ and over a cross sectional area S becomes:

$$W = \frac{1}{2}\rho S V_1^3, [\frac{Joules}{m^2.s} m^2], Watts. \tag{21}$$

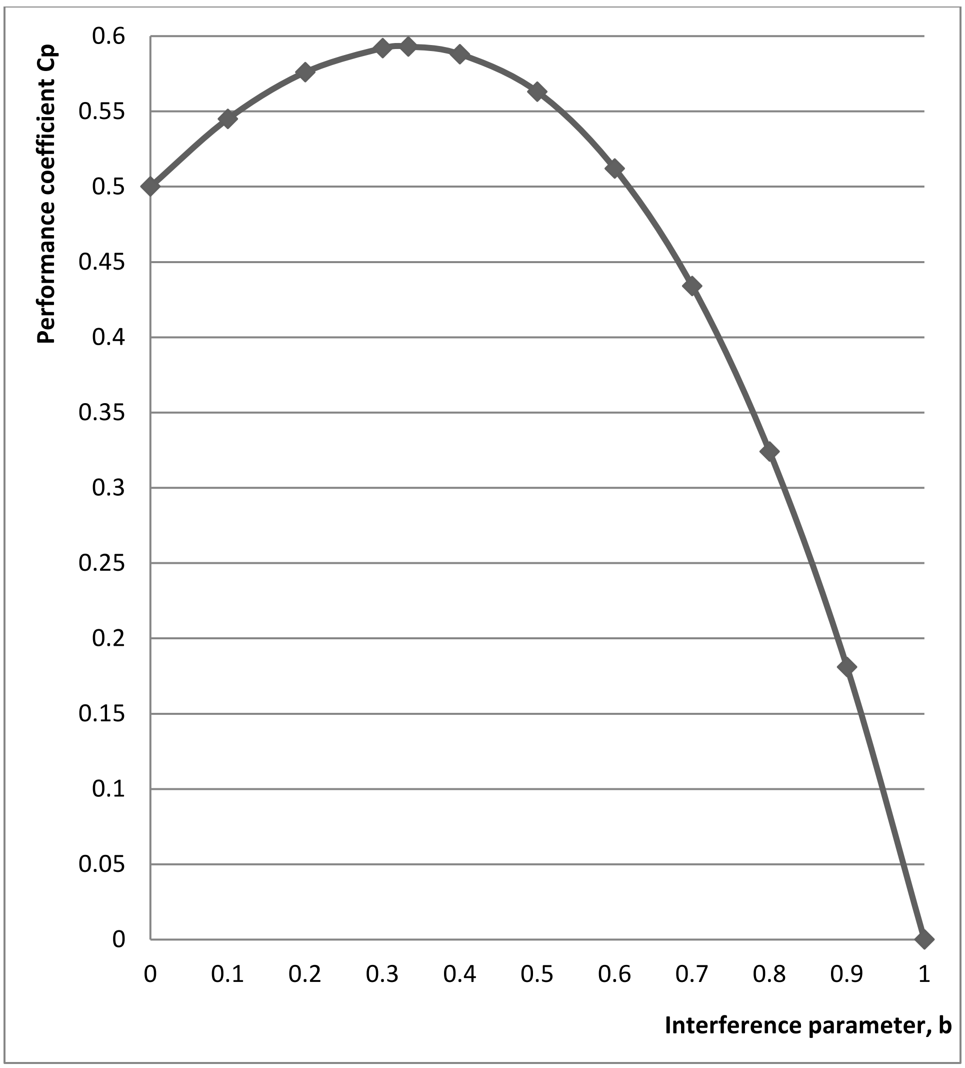
The performance coefficient, or efficiency, is the dimensionless ratio of the extractable power P to the kinetic power W available in the undisturbed stream:

$$C_p = \frac{P}{W} \tag{22}$$

The performance coefficient is a dimensionless measure of the efficiency of a wind turbine in extracting the energy content of a wind stream.

Substituting the expressions for P from Eqn. 19 and for W from Eqn. 21 we have:

$$
\begin{aligned}
C_p &= \frac{P}{W} \\
&= \frac{\frac{1}{4}\rho S V_1^3 \left(1-b^2\right)\left(1+b\right)}{\frac{1}{2}\rho S V_1^3} \\
&= \frac{1}{2}\left(1-b^2\right)\left(1+b\right)
\end{aligned}
\tag{23}
$$



C_p	0.500	0.545	0.576	0.592	0.593	0.588	0.563	0.512	0.434	0.324	0.181	0.00
b	0.0	0.1	0.2	0.3	1/3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

Figure 3. Graph of the performance coefficient C_p as a function of the interference

parameter b.

When b = 1, $V_1 = V_2$ and the wind stream is undisturbed, leading to a zero performance coefficient. When b = 0, $V_1 = 0$, the turbine stops all the air flow and the performance coefficient is equal to 0.5. It can be noticed from the graph that the performance coefficient reaches a maximum around b = 1/3.

MAXIMUM PERFORMANCE COEFFICIENT, THE BETZ'S LIMIT

A condition for maximum performance can be obtained by differentiation of Eq. 23 with respect to the interference factor b.

Applying the chain rule of differentiation:

$$\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx},$$

and equating the derivative to zero, yields:

$$\begin{aligned}
\frac{dC_p}{db} &= \frac{1}{2}\frac{d}{db}[\left(1-b^2\right)\left(1+b\right)] \\
&= \frac{1}{2}[\left(1-b^2\right) - 2b\left(1+b\right)] \\
&= \frac{1}{2}(1-b^2-2b-2b^2) \\
&= \frac{1}{2}(1-3b^2-2b) \qquad (24) \\
&= \frac{1}{2}(1-3b)(1+b) \\
&= 0
\end{aligned}$$

This equation yields a trivial solution:

$$\begin{aligned}
&(1+b) = 0 \\
&b = \frac{V_2}{V_1} = -1, \Rightarrow V_2 = -V_1
\end{aligned}$$

and a practical physical solution:

$$\begin{aligned}
&(1-3b) = 0 \\
&b = \frac{V_2}{V_1} = \frac{1}{3}, \Rightarrow V_2 = \frac{1}{3}V_1 \qquad (25)
\end{aligned}$$

meaning that for optimal operation, the downstream velocity V_2 should be equal to one third of the upstream velocity V_1.

The maximum or optimal value of the performance coefficient C_p becomes from Eqn. 23:

$$
\begin{aligned}
C_{p,opt} &= \frac{1}{2}\left(1-b^2\right)\left(1+b\right) \\
&= \frac{1}{2}\left(1-(\frac{1}{3})^2\right)\left(1+\frac{1}{3}\right) \\
&= \frac{16}{27} \\
&= 0.59259 \\
&= 59.26 \ \textit{percent}
\end{aligned}
\tag{26}
$$

This is referred to as the Betz's limit. It was first formulated in 1919, and applies to all wind energy converters designs. It is the theoretical power fraction that can be extracted from an ideal wind stream.

Modern wind machines operate at a slightly lower practical non ideal performance coefficient. It is generally reported in the range of:

$$C_{p,prac.} \approx \frac{2}{5} = 40 \ \textit{percent} \tag{27}$$

RESULT I

From Eqns. 14 and 25, there results that:

$$
\begin{aligned}
V &= \frac{1}{2}(V_1+V_2) \\
&= \frac{1}{2}(V_1+\frac{V_1}{3}) \\
&= \frac{2}{3}V_1
\end{aligned}
\tag{28}
$$

RESULT II

From the continuity Eqn. 7:

$$\dot{m} = \rho S_1 V_1 = \rho S V = \rho S_2 V_2 = \text{constant}$$

$$\text{S=S}_1 \frac{V_1}{V} = \frac{3}{2} S_1 \qquad (29)$$

$$\text{S}_2\text{=S}_1 \frac{V_1}{V_2} = 3S_1$$

This implies that the cross sectional area of the airstream downwind of the turbine expands to 3 times the area upwind of it.

POWER EXTRACTION OF THE IDEAL TURBINE CAVEAT

One can try to define the power extraction from the wind in two different ways. In the first approach, one can define the power extraction by an ideal turbine from Eqns. 28, 29 as:

$$\begin{aligned}
P_{ideal}^{1} &= P_{upwind} - P_{downwind} \\
&= \frac{1}{2}\rho S_1 V_1^3 - \frac{1}{2}\rho S_2 V_2^3 \\
&= \frac{1}{2}\rho S_1 V_1^3 - \frac{1}{2}\rho 3 S_1 (\frac{1}{3} V_1)^3 \\
&= \frac{1}{2}\rho (\frac{8}{9} S_1 V_1^3) \\
&= \frac{8}{9}\frac{1}{2}\rho S_1 V_1^3
\end{aligned}$$

This suggests that fully 8/9 of the energy available in the upwind stream can be extracted by the turbine. That is a confusing result since the upwind wind stream has a cross sectional area that is smaller than the turbine intercepted area.

The second approach yields the correct result by redefining the power extraction at the wind turbine using the area of the turbine as S = 3/2 S_1:

$$\begin{aligned}
P_{ideal} &= \frac{1}{2}\rho (\frac{8}{9} S_1 V_1^3) \\
&= \frac{1}{2}\rho (\frac{8}{9}\frac{2}{3} S V_1^3) \\
&= \frac{1}{2}\rho (\frac{16}{27} S V_1^3) \\
&= \frac{16}{27}\frac{1}{2}\rho S V_1^3
\end{aligned} \qquad (30)$$

The Betz coefficient:

$$Betz\ coefficient = \frac{16}{27} = 0.592593 = 59.26\ percent\,, \quad (31)$$

suggests that a wind turbine can extract ***at most*** 59.3 percent of the energy in an undisturbed wind stream.

Considering the frictional losses, blade surface roughness, and mechanical imperfections yields 35-40 percent of the power available in the wind under ideal conditions.

MAXIMUM POWER CONTENT, BETZ'S FORMULA

Another important perspective can be obtained by estimating the maximum power content in a wind stream. For a constant upstream velocity, we can deduce an expression for the maximum power content for a constant upstream velocity V_1 of the wind stream by differentiating the expression for the power P with respect to the downstream wind speed V_2, applying the chain rule of differentiation and equating the result to zero as:

$$\begin{aligned}
\left.\frac{dP}{dV_2}\right|_{V_1} &= \frac{1}{4}\rho S \frac{d}{dV_2}[(V_1+V_2)^2(V_1-V_2)] \\
&= \frac{1}{4}\rho S \frac{d}{dV_2}[(V_1^2-V_2^2)(V_1+V_2)] \\
&= \frac{1}{4}\rho S[(V_1^2-V_2^2)-2V_2(V_1+V_2)] \\
&= \frac{1}{4}\rho S(V_1^2-V_2^2-2V_1V_2-2V_2^2) \\
&= \frac{1}{4}\rho S(V_1^2-3V_2^2-2V_1V_2) \\
&= 0
\end{aligned} \quad (32)$$

Solving the resulting equation by factoring it:

$$\begin{aligned}
(V_1^2-3V_2^2-2V_1V_2) &= 0 \\
(V_1+V_2)(V_1-3V_2) &= 0
\end{aligned} \quad (33)$$

This equation has a first trivial solution:

$$\begin{aligned}
(V_1+V_2) &= 0 \\
V_2 &= -V_1
\end{aligned} \quad (34)$$

and a second practical physical solution:

$$(V_1 - 3V_2) = 0$$
$$V_2 = \frac{1}{3}V_1 \qquad (35)$$

This implies the simple result that that the most efficient operation of a wind turbine occurs when the downstream speed V_2 is 1/3 of the upstream speed V_1.

Adopting the second solution and substituting it in the expression for the power in Eqn. 16 we get the expression for the maximum power that could be extracted from a wind stream as:

$$\begin{aligned} P_{max} &= \frac{1}{4}\rho S\left(V_1^2 - V_2^2\right)\left(V_1 + V_2\right) \\ &= \frac{1}{4}\rho S\left(V_1^2 - \frac{V_1^2}{9}\right)\left(V_1 + \frac{V_1}{3}\right) \\ &= \frac{1}{4}\rho S V_1^3\left(1 - \frac{1}{9}\right)\left(1 + \frac{1}{3}\right) \\ &= \frac{16}{27}\frac{\rho}{2}V_1^3 S \ \ [Watt] \end{aligned} \qquad (36)$$

This expression constitutes the famous Betz's formula. Its original form is shown in Fig. 4, expressed in terms of the rotor diameter D, where the swept rotor area S is:

$$S = \frac{\pi D^2}{4} \qquad (37)$$

and:

$$P_{max} = \frac{16}{27}\frac{\rho}{2}V_1^3\frac{\pi D^2}{4} \quad [Watt] \qquad (36)'$$

$$L_{max} = \frac{16}{27} \cdot \frac{\varrho}{2} v^3 \cdot \frac{D^2\pi}{4} \ \ mkg/sek$$

Figure 4. Original form of Betz's formula. L corresponds to "Leistung" meaning "Power" in German, or in other contexts, "Supply".

```
!        Betz Equation
!        Program written in ANSI Fortran
!        Digital Visual Fortran Compiler
!        Procedure saves output to file:output1
!        This output file can be exported to a plotting routine
!        M. Ragheb, University of Illinois
!
```

```
!           pmax=(16/27)(rho/2.)(speed)**3 (pi*diameter**2)/4 [Watts]
!           pmax : Maximum extractable power from a windstream
!           rho  : air density [kg/m**3]
!           d    : rotor diameter [m**2]

            program betz
            real :: rho = 1.25
            real :: pi = 3.14159
            integer :: steps_speed=30
            integer :: steps_diameter=100
            real pmax(31,101)
            real speed, diameter

            write(*,*) x, lambda
!           Open output file
            open(10,file='output1')
!           Calculate ratio maximum power, pmax
            steps_speed = steps_speed + 1
            steps_diameter = steps_diameter +1
            do i =  1, steps_speed,5
                     speed = i - 1
                     do j =  1, steps_diameter,10
                              diameter = j-1
                              pmax(i,j) = (16./27.)*(speed**3.)*(pi*(diameter**2.)/4.)
!           Write results on output file
                              write(10,*) speed,diameter,pmax(i,j)
!           Display results on screen
                              write(*,*)  speed,diameter,pmax(i,j)
!                    pause
                     end do
            end do
            end
```

Figure 5. Fortran procedure for the estimation of Betz Equation.

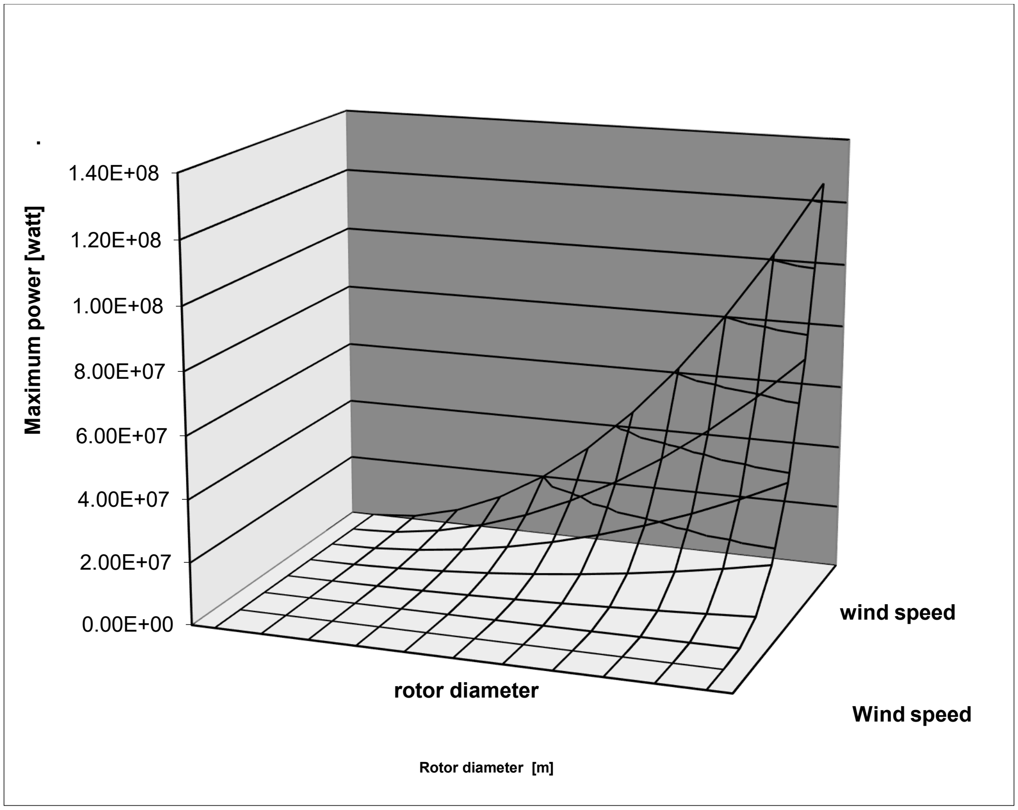


Figure 6. Maximum power as a function of the rotor diameter and the wind speed. The power increases as the square of the rotor diameter and more significantly as the cube of the wind speed.

The most important implication from Betz's formula is that there must be a wind speed change from upstream to downstream in order to extract energy from the wind, in fact by braking it using a wind energy converter.

If no change in the wind speed occurs, energy cannot be efficiently extracted from the wind. Realistically, no wind machine can totally bring the air to a total rest, and for a rotating machine, there will always be some air flowing around it. Thus a wind machine can only extract a fraction of the kinetic energy of the wind. The wind speed on the rotors at which energy extraction is maximal has a magnitude lying between the upstream and downstream wind velocities.

Betz's law reminds us of the Carnot cycle efficiency in Thermodynamics suggesting that a heat engine cannot extract all the energy from a given source of energy and must reject part of its available heat input back to the environment.

SPECIAL CASE

If we take the specific mass or density of the air as:

$$\rho = 1.25[\frac{kg}{m^3}], \tag{38}$$

a useful expression for the maximum power that can be extracted from the wind stream becomes:

$$P_{\max} = 0.3704\, S V_1^3 \,[Watt] \tag{39}$$

POWER CONTROL

INTRODUCTION

The free air stream itself is not controllable. Wind turbines themselves require either active or passive regulation or a combination of them.

Active control methods include varying the pitch of the whole blades or just the blade tips using ailerons.

Passive control results from the adoption of rigid or flexible blade profiles that produce aerodynamic stall at high wind speeds without a change of the blade pitch.

Regulation achieved by controlling the power extracted by the rotor is necessary since there is little opportunity to store excess energy within the turbine even though some short term storage exists in large machines due to the inertia of the rotor and drive-train, and small variations in the rotor speed.

PHILOSOPHY OF WIND TURBINE CONTROL

One can base the philosophy of turbine control on the following operational requirements:

1. The generation of maximum power up to the turbine's rated power.
2. The attainment of a satisfactory electrical power quality.
3. The minimization of variable and transient loads, particularly fatigue inducing variable loads thus maximizing the turbine lifetime.

The degree to which these operational objectives are attainable, through the initial design and the use of control systems is an ongoing subject for research and development.

Whether a passive or active system is used, control in some form is essential as a means of preventing turbine runaway and catastrophic failure in gusty wind conditions.

STALL REGULATION

Passive control relies on the turbine's inherent physical characteristics, where the aerodynamic properties of the rotor limit the torque produced at high wind speeds. In stall regulation, control of the rotor power is achieved by exploiting the stall characteristics of the rotor blade. A blade stalls when the laminar flow over the airfoil breaks down and the blade loses its lift. This is the same mechanism as in an aircraft wing stalling when there is no longer a sufficient lift force to counteract the gravity force. This occurs at low speeds relative to the air.

In stall control the cross section of the rotor blade is designed in such a way that at the higher wind speeds, the stall conditions occur progressively from the root side to the tip. The higher the wind speed the larger the section of the rotor blade that stalls.

The advantage of this form of regulation is the absence of moving parts and the negation of the need for an active control system.

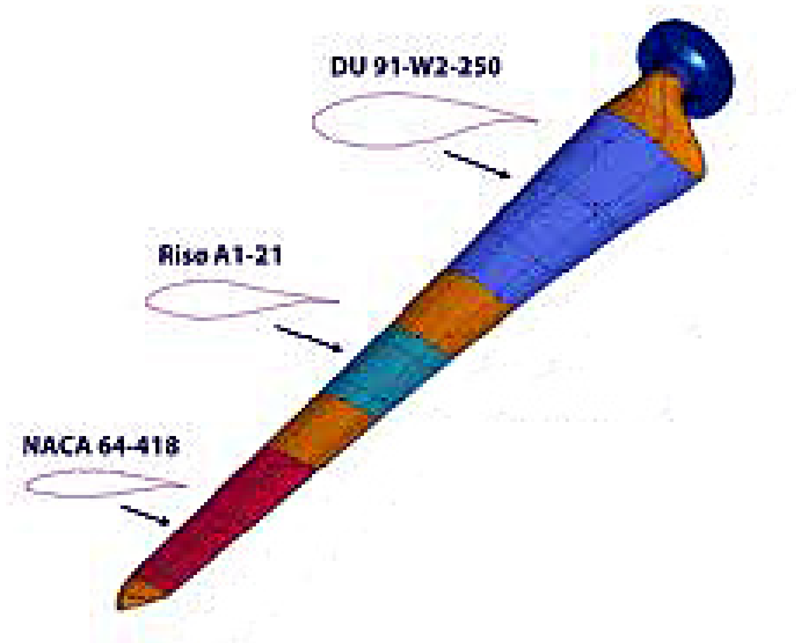


Figure 7. Variable airfoil cross section profiles of a wind rotor allows for stall control.

Even though stall regulation appears simple in principle, it presents a highly sophisticated aerodynamic design problem. Once the rotor blades have been designed and installed it becomes impossible to operate the turbine against a power set point without making other mechanical modifications. At above the rated wind speed conditions the turbine is constrained to generate the desired power level. Although power is limited in the stall regime, there still remain significant rotor, nacelle and structural tower loads due to the rotor thrust.

Due to the lack of sufficient torque at low wind speeds, the turbine may be unable to self start until the wind speed has reached a cut-in speed value, unless the generator is used as a motor to start the machine. Power generation opportunities are lost, since the highest probability of wind speed is at the lower values in the Weibull and Rayleigh probability density functions of wind speeds.

In addition, the blades or blade tips cannot be feathered, as in small wind machines in gusty wind conditions when the machine has to be shut down.

VARIABLE PITCH CONTROL

Active wind turbine control requires that either the operation of the turbine to be modified in response to a measurement of its state, or the load to be modified to match the output of the turbine to maintain the optimal rotational speed. This is designated as positive feed forward control, from rotor speed to electrical load.

The principle of negative feedback pitch control is to alter the rotor aerodynamic characteristics and so influence the developed rotor power.

There exist two ways for controlling the blade pitch in response to too much wind power, either by reducing the angular incidence of the wind onto the blade or the angle of attack or by increasing the angle of attack.

INCREASED INCIDENCE OR PITCH ASSISTED STALL METHOD

As the wind speed increases the blade control section is moved in such a way that its angle of incidence to the relative airflow increases. As a result, the rotor blade begins to stall and as a consequence the rotor power falls.

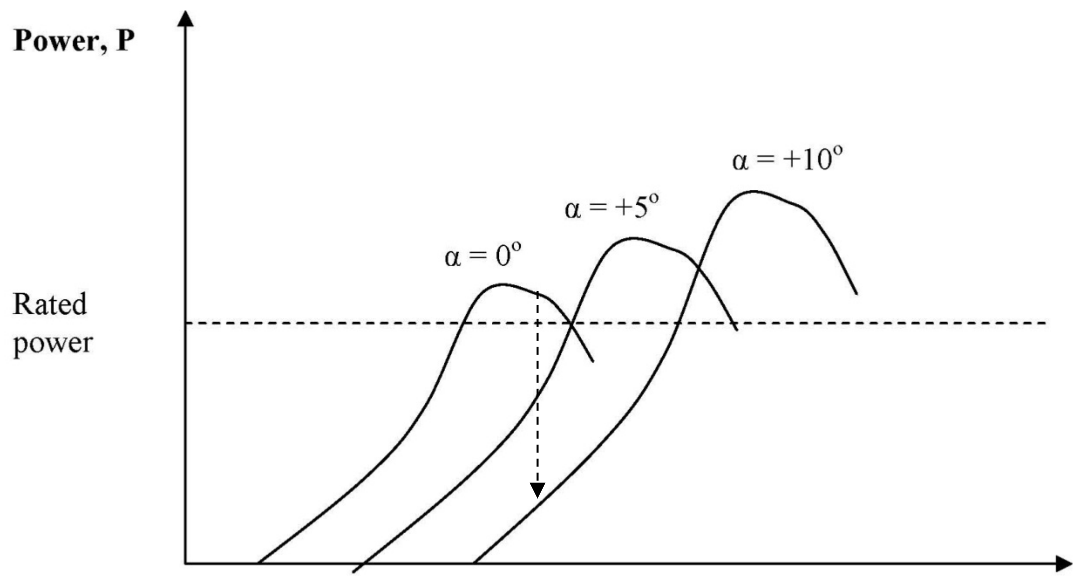


Figure 8. Pitch regulation by increasing the angle of attack.

When operating under a stall condition, the rotor loses its aerodynamic damping and that results into increased drag loads.

This approach provides good aerodynamic braking but it is not as popular as the decreasing incidence option or pitch feathering method.

DECREASED INCIDENCE OR PITCH FEATHERING METHOD

This approach is the most popular method of pitch control in modern wind machines. At higher wind speeds the control section of the blade is feathered. This reduces the rotor power available to the drive train of the turbine.

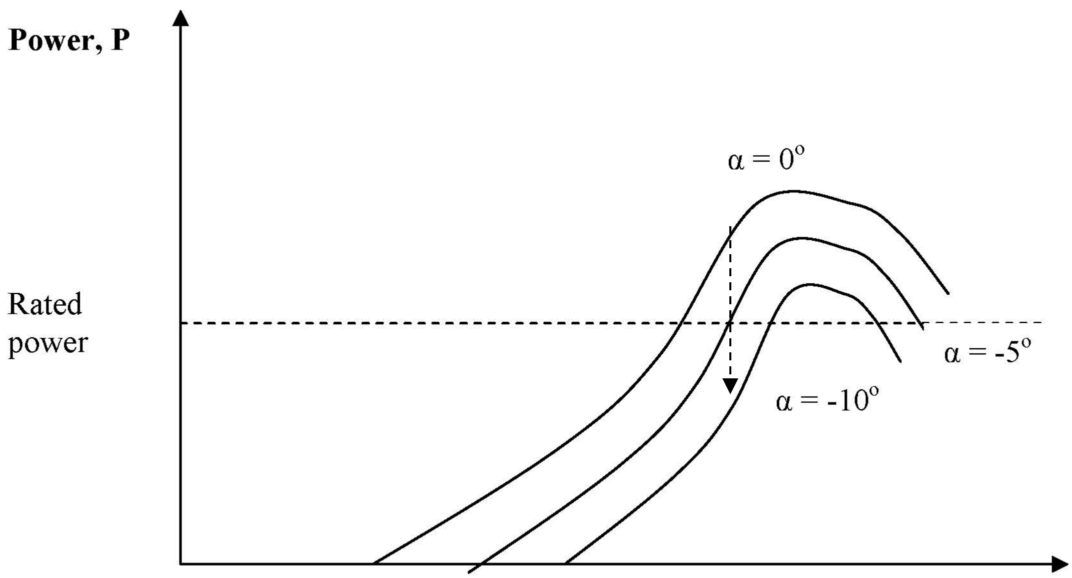


Figure 9. Pitch control by decreasing the angle of incidence, or pitch feathering.

COMPARISON OF PITCH CONTROL METHODS

The above mentioned approaches of pitch control are useful in starting the turbine. The blade pitch is adjusted to provide a high starting torque to start the rotor from rest.

The choice of the optimal amount of blade pitch control surface continues to be under investigation. Full span pitch control covering the whole surface of the rotor blade provides a larger control surface.

As the most important proportion of the rotor torque is attributable to the outer section of the blade at the tips, a partial span control surface could be just as effective. By removing the controllable surface away from the critical root area, a fixed and hence stronger blade root can be constructed.

A criticism of such a blade tip control approach is the location of the actuator mechanisms inside the blade, and its location at a distance from the hub. This requires a stiffer construction for the blade caused by the increased forces at the tip.

PITCH CONTROL LOOP

A fundamental consideration in pitch control is the bandwidth of the pitch control loop that is related to the minimum change in wind speed that results in the actuation of the pitch mechanism.

The pitch control servo mechanism should not be subjected to an excessively high frequency demand. A control action that is too responsive will induce excessive torsion blade loads. The fatigue loading implications of such loads at high magnitude and frequency can be serious.

The bandwidth of the control loop is intimately related to the drive train and the generator dynamics and is the subject of current research.

The pitch control servo mechanism can be either electrical or hydraulic. The electrical approach is similar to the modern fly by wire approach in airplane wing and rudder surfaces.

The bandwidth of the mechanism could be a limiting factor to the closed loop system bandwidth and the responsive control necessary for the load alleviation will not be achievable.

The control system bandwidth has important implications for the effect of measurement noise on the system performance. Measurement noise on the power, torque or speed signal, used for feedback, is by nature a wide band signal. The effective measurement noise spectrum, as seen by the actuator, is determined by the controller and by the system dynamics. For instance, a derivative action will increase the system bandwidth but at the expense of increased pitch actuator response to the measurement noise.

YAW CONTROL

In this method the effective rotor area in the free air stream is changed by yawing or rotating the complete turbine rotor and nacelle. By the alteration of the effective rotor area presented to the wind, the rotor power is controlled. It is believed that a passive mechanism is a preferable option on the basis of reliability.

Yawing has been a common control method for small turbines and water pumps, but has received little attention in large wind machines.

AILERON CONTROL

Aileron control is used in aircraft for takeoffs and landings. It is uncommon in wind machines, but was a basic method of control in early Danish machines built during the 1980's.

The principle of operation is to selectively alter both the lift and load forces by altering the aerodynamic characteristics of the blade airfoil. This in turn alters the aerodynamic efficiency of the rotor.

Ailerons have been suggested for speed control following the loss of load or over-speed protection. In this situation aileron control offers advantages over pitch control in that it enables the use of a smaller actuator mechanism with the rotor blade made lighter and stronger.

ELECTRICAL POWER OUTPUT

The coefficient of performance is the fraction of the wind power extracted by a wind turbine:

$$C_p = \frac{P_m}{\frac{1}{2}\rho S V_1^3} \qquad (40)$$

Thus the actual mechanical power in the rotation of the rotor blades is:

$$P_m = C_p \frac{1}{2}\rho S V_1^3 = C_p P_w \tag{41}$$

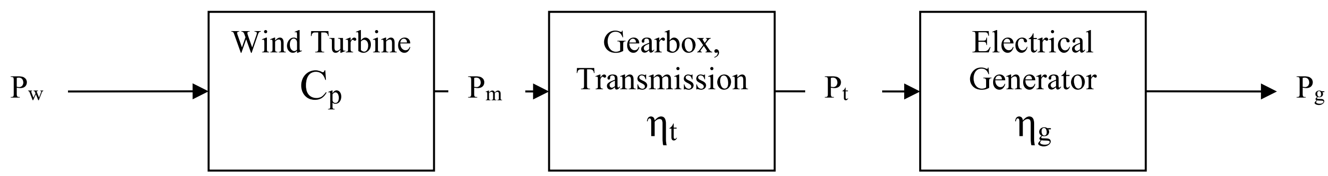


Figure 10. Wind turbine efficiencies.

The mechanical power in the rotor blades rotation is coupled to a transmission or gear box introducing a transmission or gearbox efficiency, so that the transmission power is given by:

$$P_t = C_p \eta_t \frac{1}{2}\rho S V_1^3 = C_p \eta_t P_w \tag{42}$$

The load can be now an electrical generator, a water pump, a water heater, compressor or an electrolyzer for the production of hydrogen, introducing yet another efficiency:

$$P_g = C_p \eta_t \eta_g \frac{1}{2}\rho S V_1^3 = C_p \eta_t \eta_g P_w [Watts(e)] \tag{43}$$

The actual electrical production will be further affected with the intermittency, plant or capacity factor, so that the actual electrical power generation of the wind turbine at its rated speed is:

$$P_e = CF.C_p \eta_t \eta_g \frac{1}{2}\rho S V_1^3 = CF.C_p \eta_t \eta_g P_w [Watts(e)] \tag{44}$$

Only the last two equations will give the electrical power production of a wind turbine. The same wind turbine can be attached to different transmission and generator sizes producing different electrical power levels. A higher price for the same wind turbine could be obtained by a vendor from an uniformed buyer, if the price is quoted in terms of dollars per unit power rating.

DISCUSSION

For simplicity of construction, passive regulation is a desirable feature. However, due to a number of operational considerations, as well as the presence of uncertainties in the aerodynamic behavior of large stall regulated rotors, active regulation is favored by many designers and builders. Active control is expected to be used for the foreseeable future and there exists an urgent need to clarify and optimize its role in wind power generation.

Current research into stall regulated blades hopes to extend traditional airfoil theory into the stall region and hence to obtain more information about the stall regulation process.

Aileron, stall and yaw control in large machines need further consideration.

The power control of wind machines is a subject of ongoing research and development. To suppress the random variations in power generation one can propose the use of various methods of energy storage. In the early steam engines, flywheels were used to smooth out the power output. The use of flywheels and their equivalents as superconducting magnets or the conversion of the electrical power into hydrogen from water electrolysis are energy storage alternatives worthy of consideration.

Anticipatory or predictive control rather than reactive control needs to be developed in a wind farm environment in view of optimal operation and the prevention of occurrence of multiple fault conditions under unfavorable wind situations.

EXERCISES

1. By differentiation the expression of the power coefficient:

$$C_p = \frac{P}{W} = \frac{1}{2}\left(1-b^2\right)\left(1+b\right)$$

with respect to the interference factor b, determine the value of the Betz' limit for wind machines.
Explain its physical meaning.

2. By differentiating the expression for the power in a wind stream:

$$P = \frac{1}{4}\rho S\left(V_1+V_2\right)^2\left(V_1-V_2\right)$$

with respect to the downstream velocity V_2 for a constant upstream velocity V_1, derive Betz's Equation for the maximum amount of power extractable from a wind stream.
Compare your result to the original equation introduced by Betz.