OPTIMAL ROTOR TIP SPEED RATIO

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INTRODUCTION

A rotor that rotates slowly will allow the wind to pass unperturbed through the gaps between the blades. A rotor rotating rapidly will appear as a solid wall to the wind. It is necessary in the design of wind turbines to match the angular velocity of the rotor to the wind speed in order to obtain maximum or optimal rotor efficiency.

If the rotor of the wind turbine turns too slowly, most of the wind will pass undisturbed through the openings between the blades with little power extraction. On the other hand, if the rotor turns too fast, the rotating blades act a solid wall obstructing the wind flow, again reducing the power extraction.

Wind turbines must thus be designed to operate at their optimal wind tip speed ratio in order to extract as much power as possible from the wind stream. Wind tip ratios depend on the particular wind turbine design used, the rotor airfoil profile used, as well as the number of used blades.

For grid connected wind turbines with three rotor blades the optimal wind tip speed ratio is reported as 7, with values over the range 6-8.

EFFECT OF ROTOR TIP SPEED RATIO

The choice of the tip speed ratio for a particular wind turbine design depends on several factors.

In general a high tip speed ratio is a desirable feature since it results in a high shaft rotational speed that is needed for the efficient operation of an electrical generator. A high tip speed ratio however entails several possible disadvantages:

1. Rotor blade tips rotating at a speed larger than 80 m/sec will be subject to erosion of the leading edges from their impact with dust or sand particles in the air, and will require the use of special erosion resistant coatings much like in the design of helicopter blades.
2. Noise generation in the audible and non audible ranges.
3. Vibration, particularly in the cases of two or single bladed rotors.
4. Starting difficulties if the shaft is stiff to start rotation.
5. Reduced rotor efficiency due to drag and tip losses.
6. Excessive rotor speeds would lead to a runaway turbine, leading to its catastrophic failure, and even disintegration.

TIP SPEED RATIO, TSR

The relationship between the wind speed and the rate of rotation of the rotor is characterized by a non-dimensional factor, known as the Tip Speed Ratio (TSR) or lambda:

$$\text{Tip speed ratio: } \lambda = \frac{\text{speed of rotor tip}}{\text{wind speed}} = \frac{v}{V} = \frac{\omega r}{V} \qquad (1)$$

where:

V	is the wind speed [m/sec]
$v = \omega \mathrm{r}$	is velocity of rotor tip [m/sec]
r	is rotor radius [m]
$\omega = 2\pi f$	is the angular velocity [radian/sec]
f	is the frequency of rotation [Hz], [sec^{-1}]

This dimensionless factor arises from the detailed treatment of the aerodynamic theory of wind power extraction.

EXAMPLE

At a wind speed of 15 m/sec, blade radius of 10 m, rotating at 1 rotation per second:

$$f = 1[\frac{\text{rotation}}{\text{sec}}],$$
$$\omega = 2\pi f = 2\pi[\frac{\text{radian}}{\text{sec}}]$$
$$v = \omega r = 2\pi.10 = 20\pi[\frac{\text{m}}{\text{sec}}]$$
$$\lambda = \frac{\omega r}{V} = \frac{20\pi}{15} = \frac{62.83}{15} = 4$$

EXAMPLE

The Suzlon S.66/1250, 1.25 MW rated power at 12 m/s rated wind speed wind turbine design has a rotor diameter of 66 meters and a rotational speed of 13.9-20.8 rpm. Its angular speed range is:

$$\omega = 2\pi f$$
$$= 2\pi \frac{13.9 - 20.8}{60}[\text{radian}.\frac{\text{revolutions}}{\text{minute}}.\frac{\text{minute}}{\text{second}}]$$
$$= 1.46 - 2.18[\frac{\text{radian}}{\text{sec}}]$$

The range of its rotor’s tip speed can be estimated as:

$$
\begin{aligned}
v &= \omega r \\
&= (1.46 - 2.18)\frac{66}{2} \\
&= 48.18 - 71.94[\frac{\text{m}}{\text{sec}}]
\end{aligned}
$$

The range of its tip speed ratio is thus:

$$
\begin{aligned}
\lambda &= \frac{\omega r}{V} \\
&= \frac{48.18 - 71.94}{12} \\
&\simeq 4 - 6
\end{aligned}
$$

OPTIMAL ROTOR TIP SPEED RATIO

The optimal tip speed ratio for maximum power extraction is inferred by relating the time taken for the disturbed wind to reestablish itself t_w to the time taken for a rotor blade of rotational frequency ω to move into the position occupied by its predecessor t_s .

For an n bladed rotor, the time period for the blade to move to its predecessor's position is given by:

$$
t_s = \frac{2\pi}{n\omega}[\text{sec}] \tag{2}
$$

If the length of the strongly disturbed air stream upwind and downwind of the rotor is s, then the time period for the wind to return to normal is given by:

$$
t_w = \frac{s}{V}[\text{sec}] \tag{3}
$$

If $t_s > t_w$, then some wind is unaffected. If $t_w > t_s$, then some wind is not allowed to flow through the rotor. The maximum power extraction occurs when these two time periods are about equal:

$$
\begin{aligned}
&t_s \approx t_w \\
&\frac{2\pi}{n\omega} \approx \frac{s}{V} \Rightarrow \frac{n\omega}{V} \approx \frac{2\pi}{s}
\end{aligned} \tag{4}
$$

From which the optimal rotational frequency is:

$$
\omega_{opt} \approx \frac{2\pi V}{ns} \tag{5}
$$

Consequently, for optimal power extraction, the rotor blade must rotate at a rotational frequency that is related to the speed of the incoming wind. This rotor rotational frequency decreases as the radius of the rotor increases and can be characterized by calculating the optimal tip ratio as:

$$\lambda_{opt} \approx \frac{\omega_{opt} r}{V} \approx \frac{2\pi}{n}\left(\frac{r}{s}\right) \tag{6}$$

EFFECT OF THE NUMBER OF ROTOR BLADES

The optimal tip speed ratio depends on the number of rotor blades n of the wind turbine. The smaller the number of blades, the faster the wind turbine has to rotate to extract maximum power from the wind.

For an n bladed machine it has been empirically observed that s is equal to about half a rotor radius or:

$$\frac{s}{r} \approx \frac{1}{2}$$

or the ratio (s/r) is approximately equal to 0.5, thus we can write:

$$\lambda_{opt} \approx \frac{2\pi}{n}\left(\frac{r}{s}\right) \approx \frac{4\pi}{n} \tag{7}$$

For n = 2, a two bladed rotor, the maximum power extracted from the wind at $C_{p,max}$ occurs at:

$$\lambda_{opt} \approx \frac{4\pi}{2} \approx 2\pi \approx 6.283,$$

whereas for an n = 3 bladed rotor it is a lower value of:

$$\lambda_{opt} \approx \frac{4\pi}{3} \approx 1.33\pi \approx 4.19,$$

and for an n = 4 bladed rotor it is a further lower value of:

$$\lambda_{opt} \approx \pi \approx 3.14159.$$

If the aerofoil is designed with care, the optimal tip speed ratios may be about 25-30 percent above these optimal values. These highly efficient aerofoil rotor blade designs increase the rotational speed of the blade rotor therefore generating more power.

A typical three bladed rotor design would have a tip speed ratio of:

$$\lambda_{opt} \approx \frac{4\pi}{3}(1.25 - 1.30) \approx 5.24 - 5.45$$

If poorly designed blades are used resulting in a tip speed ratio that is too low, the wind turbine would have a tendency to slow and to stall.

If the tip speed ratio is too high, the turbine will rotate very fast through turbulent air, and the power will not be only optimally extracted from the wind stream, but the turbine will be highly stressed at the risk of catastrophic failure.

POWER COEFFICIENT, C_p

The power generated by the kinetic energy of a free flowing wind stream is given by:

$$P = \frac{1}{2}\rho S V^3 \ [Watt] \tag{8}$$

The cross sectional area S of the turbine in terms of its blade radius R is given by:

$$S = \pi R^2 [m^2] \tag{9}$$

From which the power P becomes:

$$P = \frac{1}{2}\rho \pi R^2 V^3 \tag{10}$$

The power coefficient is defined as the power extracted by the turbine relative to that available in the wind stream:

$$C_p = \frac{P_t}{P} = \frac{P_t}{\frac{1}{2}\rho \pi R^2 V^3} \tag{11}$$

The maximum achievable power factor is 59.26 percent, and is designated as the Betz limit. In practice, values of obtainable power coefficients are in the range of 45 percent. This value below the theoretical limit is caused by the inefficiencies and losses attributed to different configurations, rotor blades and turbine designs.

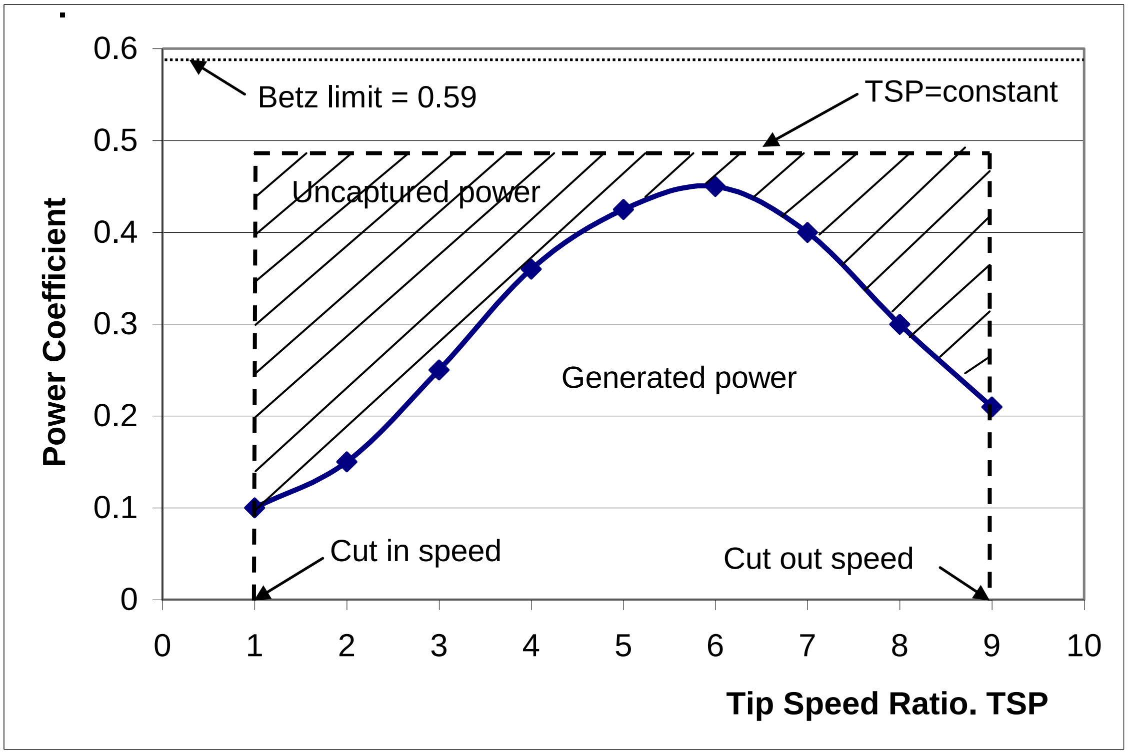

Figure 1. Power coefficient as a function of tip speed ratio for a two bladed rotor. Maximum power extraction occurs at the optimal tip speed ratio. The uncaptured power is caused by the fact that the tip speed ratio is not constant as well as the inherent inefficiencies and losses in different turbine designs.

INEFFICIENCIES AND LOSSES

The inefficiencies and losses encountered in the operation of wind turbines include the blade number losses, whirlpool losses, end losses and the airfoil profile losses.

AIRFOIL PROFILE LOSSES

The slip or slide number s is the ratio of the uplift force coefficient of the airfoil profile used C_L to the drag force coefficient C_D is:

$$s = \frac{C_L}{C_D} \tag{12}$$

Accounting for the drag force can be achieved by using the profile efficiency that is a function of the slip number s and the tip speed ratio λ as:

$$\eta_{profile} = \frac{s-\lambda}{s} = 1 - \frac{\lambda}{s} \tag{13}$$

ROTOR TIP END LOSSES

At the tip of the rotor blade an air flow occurs from the lower side of the airfoil profile to the upper side.

This air flow couples with the incoming air flow to the blade. The combined air flow results in a rotor tip end efficiency, $\eta_{\text{tip end}}$.

WHIRLPOOL LOSSES

In the idealized derivation of Betz' law, the wind does not change its direction after it encounters the turbine rotor blades. In fact, it does change its direction after the encounter.

This is accounted for by a modified form of the power coefficient known as the Schmitz power coefficient $C_{pSchmitz}$ if the same airfoil design is used throughout the rotor blade.

Table 1. Whirlpool losses Schmitz power coefficient as a function of the tip speed ratio.

Tip speed ratio TSR λ	Whirpool Schmitz Power Coefficient $C_{pSchmitz}$
0.0	0.000
0.5	0.238
1.0	0.400
1.5	0.475
2.0	0.515
2.5	0.531
3.0	0.537
3.5	0.538
4.0	0.541
4.5	0.544
5.0	0.547
5.5	0.550
6.0	0.553
6.5	0.556
7.0	0.559
7.5	0.562
8.0	0.565
8.5	0.568
9.0	0.570
9.5	0.572
10.0	0.574

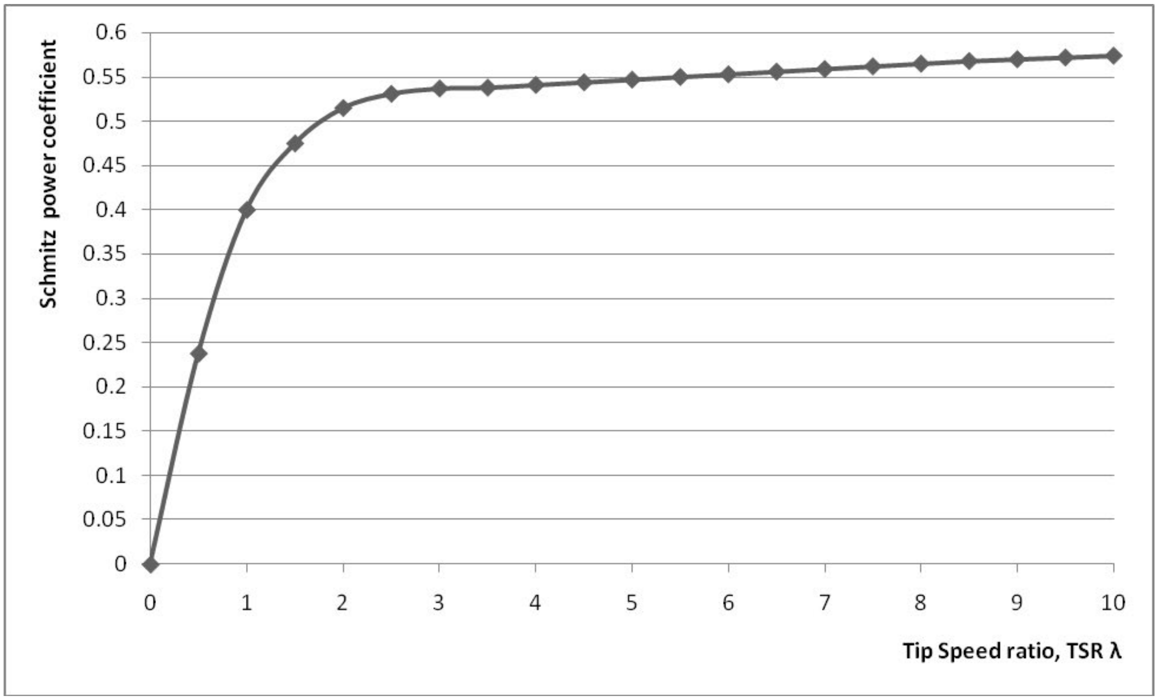

Figure 2. Schmitz power coefficient as a function of the tip speed ratio, TSR.

ROTOR BLADE NUMBER LOSSES

A theory developed by Schmitz and Glauert applies to wind turbines with four or less rotor blades. In a turbine with more than four blades, the air movement becomes too complex for a strict theoretical treatment and an empirical approach is adopted. This can be accounted for by a rotor blades number efficiency η_{blades}.

In view of the associated losses and inefficiencies, the power coefficient can be expressed as:

$$C_p \approx C_{pSchmitz} \eta_{profile} \eta_{tip\,end} \eta_{blades} \tag{14}$$

There are still even more efficiencies involved:

1. Frictional losses in the bearings and gears: $\eta_{friction}$
2. Magnetic drag and electrical resistance losses in the generator or alternator: $\eta_{electrical}$.

$$C'_p \approx C_{pSchmitz} \eta_{profile} \eta_{tip\,end} \eta_{blades} \eta_{friction} \eta_{electrical} \tag{15}$$

In the end, the Betz limit is an idealization and a design goal that designers try to reach in a real world turbine. A C_p value of 35 percent is a realistic design goal for a workable wind turbine. This is still reduced by an intermittency factor accounting for the periods of wind flow as the intermittency factor.

TIP SPEED RATIOS OF DIFFERENT DESIGNS

The theoretical maximum efficiency of a wind turbine is given by the Betz limit around 59 percent. Practically, wind turbines operate below the Betz limit. In Fig. 1 for a two bladed turbine, if it is operated at the optimal tip speed ratio of 6, its power coefficient would be around 0.45. At the cut-in speed, the power coefficient is just 0.10, and at the cut-out speed it is 0.22. This suggests that for maximum power extraction a wind turbine should be operated around its optimal wind tip ratio.

Modern horizontal axis wind turbine rotors consist of two or three thin blades and are designated as low solidity rotors. This implies a low fraction of the area swept by the rotors being solid. Its configuration results in an optimum match to the frequency requirements of modern electricity generators and also minimizes the size of the gearbox required as well as increases efficiency.

Such an arrangement results in a relatively high tip speed ratio in comparison with rotors with a high number of blades such the highly successful American wind mill used for water pumping in the American West and all over the world. The latter required a high starting torque.

The relationship between the rotor coefficient C_p and the tip speed ratio is shown for different types of wind machines. It can be noticed that it reaches a maximum at different positions for different machine designs.

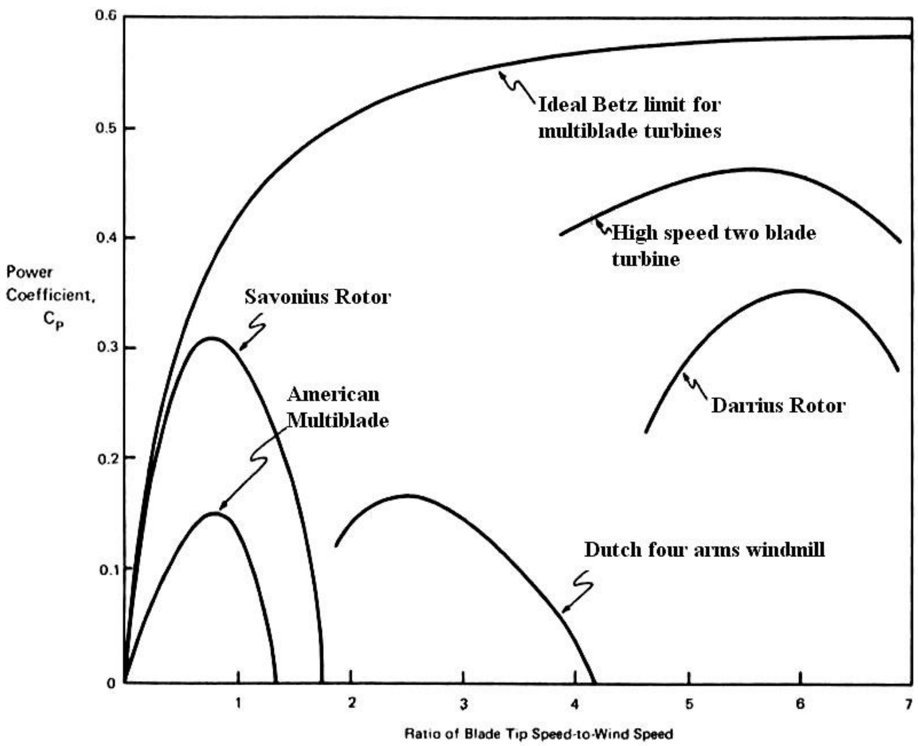

Figure 3. The power coefficient C_p as a function of the tip speed ratio for different wind machines designs. Note that the efficiency curves of the Savonius and the American multi-blade designs were inadvertently switched in some previous publications, discouraging the study of the Savonius design.

The maximum efficiencies of the two bladed design, the Darrieus concept and the Savonius reach levels above 30 percent but below the Betz limit of 59 percent.

The American multibladed design and the historical Dutch four bladed designs peak at 15 percent. These are not suited for electrical generation but are ideal for water pumping.

DISCUSSION

Wind turbines must be designed to operate at their optimal wind tip speed ratio in order to extract as much power as possible from the wind stream.

When a rotor blade passes through the air stream it leaves a turbulent wake in its path. If the next blade in the rotating rotor arrives at the wake when the air is still turbulent, it will not be able to extract power from the wind efficiently, and will be subjected to high vibration stresses. If the rotor rotated slower, the air hitting each rotor blade would no longer be turbulent. This is another reason for the tip speed ratio to be selected so that the rotor blades do not pass through turbulent air.

EXERCISE

1. For a wind speed of 15 m/s and a 3 bladed rotor radius of 10 meters rotating at 1 rotation / sec, calculate:
    1. The angular rotational frequency,
    2. The rotor tip speed,
    3. The tip speed ratio.

Compare this value to the optimal tip speed ratio.
Repeat the comparison for a 2 bladed turbine.

REFERENCES

1. N. S. Çetin, M. A Yurdusev, R. Ata and A. Özdemir, “Assessment of Optimum Tip Speed Ratio of Wind Turbines,” Mathematical and Computational Applications, Vol. 10, No.1, pp.147-154, 2005.
2. E. Hau, “Windkraftanlangen,” Springer Verlag, Berlin, Germany, pp. 110-113, 1996.
3. B. Jones, “Elements of Aerodynamics,” John Wiley and Sons, New York, USA, pp. 73-158, 1950.