SUBCRITICAL ASSEMBLIES THEORY

5/27/2013

INTRODUCTION

Experiments are needed to confirm the calculations of reactor critical sizes whether the configuration is homogeneous or heterogeneous. A subcritical assembly is usually used for this purpose. A system consisting of fuel and a moderator is gradually built up until it approaches the critical dimensions, but not until it becomes actually critical.

At the center of the assembly, a neutron source could be placed. The source neutrons are multiplied by the fissile material, and detectors are used to count them. A steady state neutron flux will be attained as long as the source is present.

SUBCRITICAL NEUTRON MULTIPLICATION

The subcritical multiplication M is defined as:

$$M=\frac{\text{Total neutron flux due to primary source and fissions}}{\text{Neutron flux due to source alone}} \tag{1}$$

To study M let us assume the following:

1. Neutron multiplying medium.
2. Multiplication factor is less than unity corresponding to subcritical multiplication:

$$k_{eff} < 1. \tag{2}$$

3. A source of neutrons: cosmic rays, spontaneous fissions, (α, n) or Po-Be or Ra-Be source is inserted in its center.
4. A control rod can be withdrawn to increase the value of k_{eff}. The effective multiplication factor for a particular assembly under consideration depends upon its size as well as its composition and arrangement.

The number of neutrons after (m-1) generations or neutron lifetimes is:

$$\begin{aligned} n &= n_0 + k_{eff} n_0 + k_{eff}^2 n_0 + ... + k_{eff}^{m-1} n_0 \\ &= n_0 (1 + k_{eff} + k_{eff}^2 + ... + k_{eff}^{m-1}) \\ &= n_0 \frac{1-k_{eff}^m}{1-k_{eff}}, m > 0. \end{aligned} \tag{3}$$

After a sufficiently long time:

$$m \rightarrow \infty,$$

and:

$$n \simeq n_0 \frac{1}{1-k_{eff}} \tag{4}$$

The subcritical multiplication is thus:

$$M = \frac{n}{n_0} = \frac{1}{1-k_{eff}} \tag{5}$$

When k_{eff} = 0.5, the subcritical multiplication levels off after several neutron lifetimes to a value:

$$M = \frac{n}{n_0} = \frac{1}{1-0.5} = 2.0$$

As k_{eff} approaches unity, the subcritical multiplication factor approaches infinity and the number of neutrons in the medium rises as a straight line with time:

$$M = \frac{n}{n_0} = 1 + 1^2 + 1^3 + \ldots + 1^{m-1} = m \tag{6}$$

As the subcritical multiplication becomes higher and higher, more time is taken by the medium to settle at a given level. Finally it does not settle out but continues to rise linearly. If the assembly were supercritical, it would increase exponentially instead.

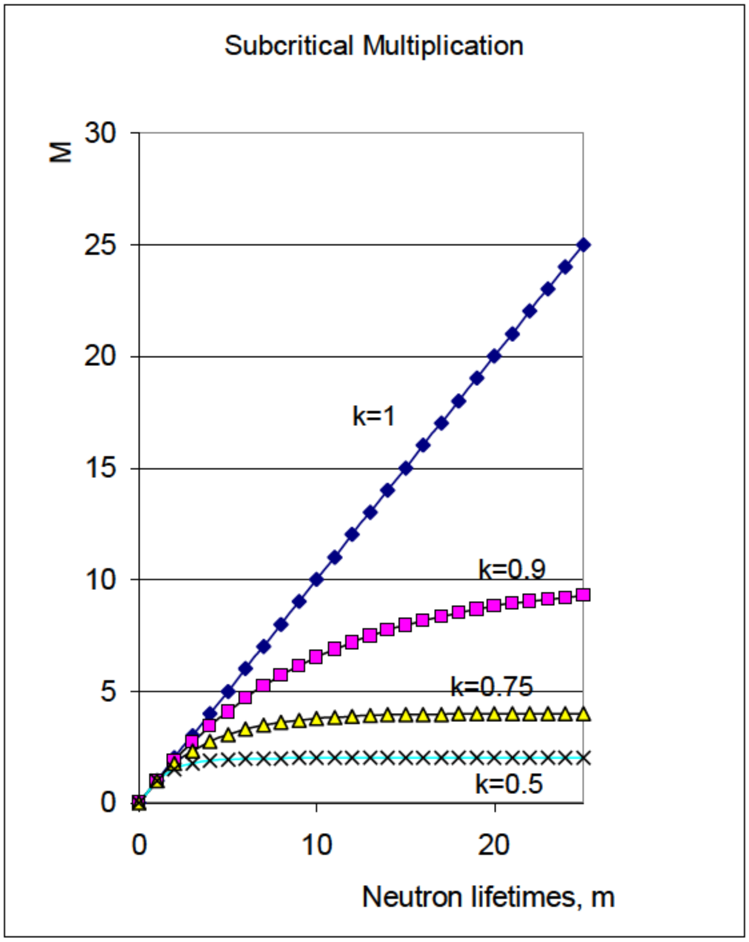

Figure 1. Subcritical multiplication as a function of the neutron lifetimes. k = effective multiplication factor.

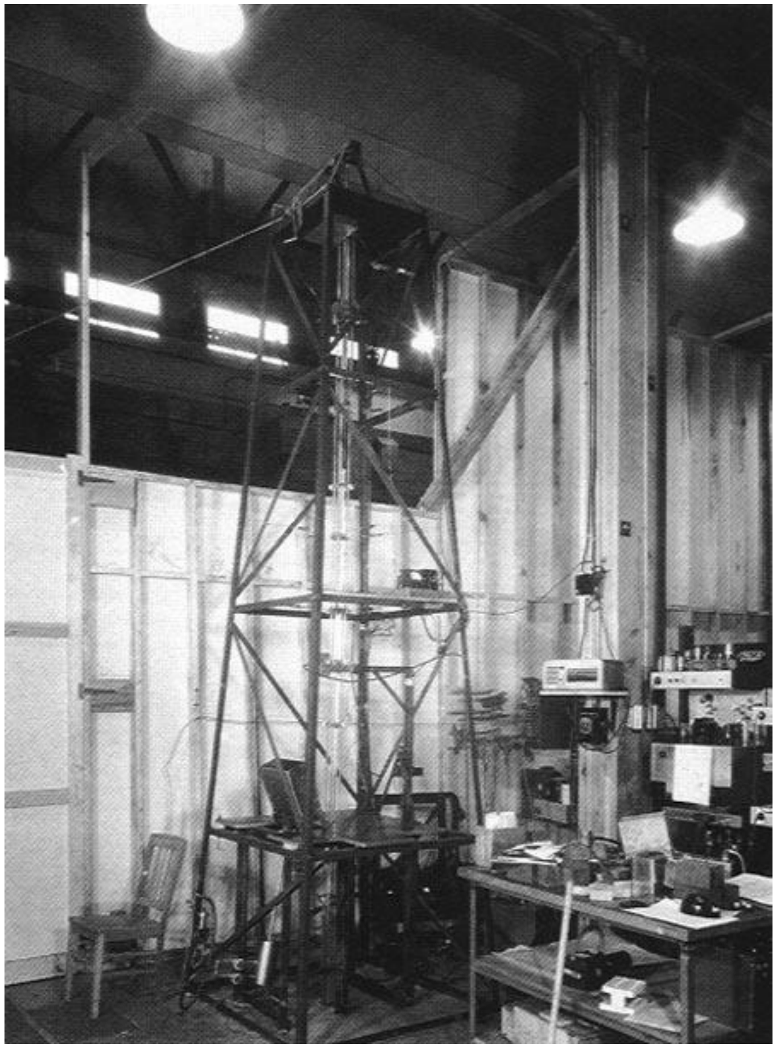

Figure 2. Guillotine setup to test the criticality of subcritical and critical configurations.

EXPERIMENTAL DETERMINATION OF SUBCRITICAL MULTIPLICATION

The neutron flux is measured in the assembly at a certain distance from the source.

The measurement is first made without the fuel, with the source and the moderator. The fuel is then added, and the measurement made at the same position. The ratio of the two values gives the required multiplication.

The observations are repeated for assemblies of different sizes approaching the critical configuration.

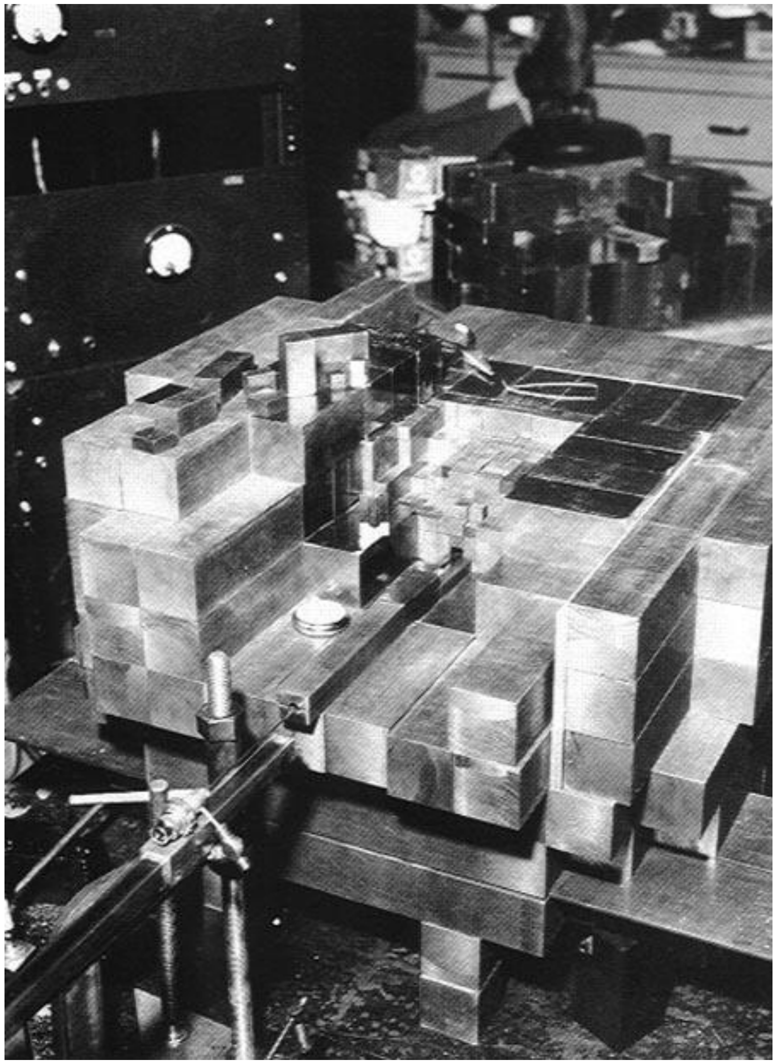

Figure 3. Beryllium blocks used as neutron reflectors for a central core in a subcritical assembly.

DETERMINATION OF CRITICAL SIZE

The reciprocal of the multiplication is plotted against the mass of the fuel or the thickness of the reflector, or the size of the core, whichever parameter is being varied.

The extrapolation of the inverse multiplication 1/M to zero gives the critical value. In this case the inverse multiplication is determined through experimental measurements and is defined as:

$$\frac{1}{M} = \frac{\text{Counting rate without fissile material}}{\text{Counting rate with fissile material}} \tag{7}$$

One can consider the determination of the critical height of an Oralloy (93.2 enriched U^{235}, ρ=18.8 gm/cm^3) cylinder. A lead reflector is kept constant and the cylinder height was incrementally increased. One could also have kept the core radius constant and increased the reflector thickness. Care must be done not to reach criticality. The counting from two different detectors generates two different curves. The critical core radius or the critical reflector thickness is obtained through extrapolation of the generated graphs at their intersection with the x-axis.

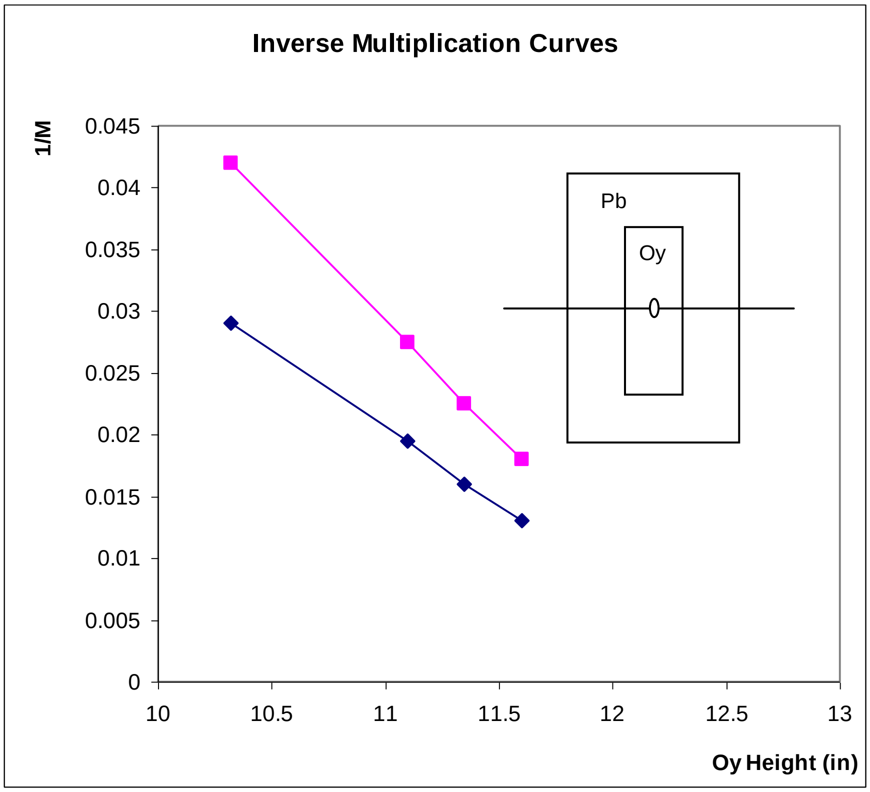

Figure 4. Extrapolation of inverse multiplication (1/M) for 9.9 cm diameter Oralloy cylinder with a 13.3 cm surrounding Pb reflector. Critical height extrapolates to 12.7 inches.

ASSEMBLY MACHINE

The experimental determination of critical masses is done on an assembly machine using either a hydraulic ram or railing along which the subcritical parts of the mass are remotely moved and added in small increments. The system is assembled remotely from a control room by raising or lowering the ram.

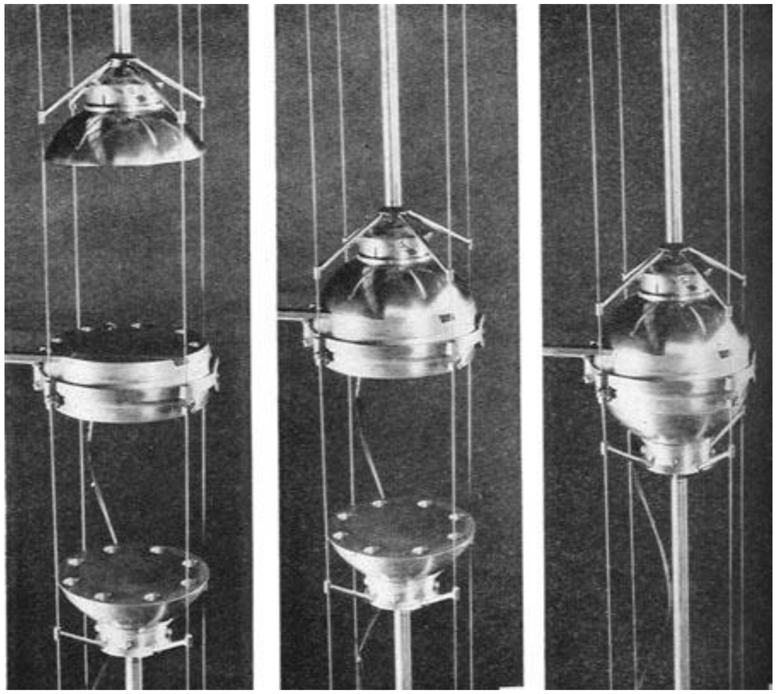

Figure 5. Parts of the Jezebel plutonium critical assembly sliding on wires.

The neutrons are counted using several detectors such as ionization chambers, lithium iodide (LiI) crystal detectors or other neutron detectors. The counter signals, in addition to being fed to scalers by conventional means, are fed to counting rate meters, which automatically separate the assembly if the counting rate exceeds a predetermined safe value.

Figure 6. Modern Assembly machine at Los Alamos National Laboratory conducting criticality studies on a Neptunium^{237} reflected with shells of highly enriched U^{235}.

A modern assembly machine consists of an 18 mil diaphragm supported above a vertical hydraulic ram. The system could be assembled remotely from a control room by raising the ram. Reflector shells can be added in small increments.

By making observations at two or more positions using different counters, several sets of data are obtained. The curves should thus extrapolate to the same points.

The size of the critical reactor can thus be determined from measurements with subcritical assemblies. A final check can then be made by constructing the critical reactor and sustaining a chain reaction in it.

Table 1 shows the results for a cylindrical assembly made of Oralloy which is 93.2 percent enriched U^{235} with a density of 18.8 [gm/cm^3], and reflected with Pb. The Pb reflector thickness was kept constant and the cylinder height was increased.

It also shows the results for a spherical assembly of 93.1 percent enriched Oralloy with a density of 18.7 [gm/cm3]. The core radius is kept a constant, and the Pb reflector thickness was varied.

Table 1. Critical Dimensions results for Oralloy reflected with lead sphere and cylinder.

Oralloy Cylinder				
Oralloy Diameter [cm]	Pb reflector thickness [cm]		Extrapolated Oralloy critical mass [kg]	Root mean square spread in mass
	Sides	Ends		
9.88	13.3	0.0	63.9	1.6 %
11.17	12.7	0,0	41.8	0.3 %
9.88	13.3	13.3	57.8	0.8 %
11.17	12.7	12.7	34.7	0.6 %

Oralloy Sphere			
Oralloy Diameter [cm]	Oralloy mass [kg]	Extrapolated Pb critical thickness [cm]	Root mean square spread in thickness
14.22	27.99	17.22	2.5 %
14.90	32.65	8.99	0.3 %

STABILITY CONSIDERATIONS

According to Eqn. 3, a reactor that has $k_{eff} = 1$ is unstable since this leads to an infinite multiplication. In fact this true if an external source is present. For stability, k_{eff} must be less than unity, but by a very small amount as shown below.

Consider a source of strength:

$$n_0 = 10^7 [\frac{neutrons}{\sec}],$$

in a reactor with a volume:

$$V = 10^6 [cm^3].$$

This leads to a volumetric source strength of:

$$S_v = \frac{n_0}{V} = \frac{10^7}{10^6} = 10[\frac{\text{neutrons}}{\text{cm}^3.\text{sec}}]$$

The neutron density in a typical thermal reactor is;

$$N = \frac{\varphi_{th}}{v_{th}} = \frac{2x10^{13}}{2x10^5} \frac{neutrons/(cm^2.\text{sec})}{cm/\text{sec}} \quad 10^8[\frac{neutrons}{cm^3}]$$

Thus, the neutrons are produced in the reactor at the rate of:

$$S_v' = \frac{N}{\ell} \simeq \frac{10^8}{10^{-3}} \simeq 10^{11}[\frac{\text{neutrons}}{\text{cm}^3.\text{sec}}]$$

where ℓ is the thermal neutron lifetime for a graphite and natural uranium system.
The neutron multiplication in this case is:

$$M = \frac{S_v'}{S_v} \simeq \frac{10^{11}}{10} \simeq 10^{10}$$

Substituting in Eqn. 5, we get:

$$M = \frac{1}{1-k_{eff}} \Rightarrow k_{eff} = 1 - \frac{1}{M} \simeq 1 - 10^{-10} \qquad (8)$$

which is infinitesimally less than unity.

CONCLUSION

In the presence of an external source, the reactor's effective multiplication factor k_{eff} is infinitesimally less than unity, for a stable system. In the absence of an external source, a steady state can only be maintained if k_{eff} is exactly unity.

EXERCISES

1. Generate the graph for $M=n/n_0$ as a function on the number of neutron lifetimes m, for the following values of k_{eff}: 1, 0.9, 0.75, 0.5.
2. Extrapolate the curves in the inverse multiplication curve for the Pb reflected Oralloy cylinder to determine its critical height in inches.

APPENDIX

Procedure to calculate subcritical multiplication in free format Fortran.

```
!       Subcritical multiplication
!       M=(1-keff**m)/(1-keff),m>0, keff<1
!       M=m, keff=1
!       Procedure saves output to file:output1
!       This output file can be exported to a plotting routine
!       M. Ragheb, University of Illinois
        program subcritical_multiplication
        integer :: steps=25
        real mul(26,4),keff(4),m(26)
        keff(1)=1.0
        keff(2)=0.9
        keff(3)=0.75
        keff(4)=0.5
        write(*,*) keff
!       Open output file
        open(10,file='output1')
!       Calculate Multiplication: M

!       Critical system keff=1.0
        steps = steps + 1
        do i =  1, steps
                m(i) = i - 1
                mul(i,1) = m(i)
!       Display results on screen
                write(*,*) m(i), mul(i,1)
!               pause
        end do
!       Subcritical System
        do j=2,4
                do i =  1, steps
                m(i) = i - 1
                mul(i,j) = (1-keff(j)**m(i))/(1-keff(j))
!       Display results on screen
                write(*,*) m(i), mul(i,j)
!               pause
                end do
        end do
!   Write final result
        do i =  1, steps
!       Write results on output file
        write(10,*) m(i), mul(i,1),mul(i,2),mul(i,3),mul(i,4)
        end do
        end
```