POINT REACTOR KINETICS

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INTRODUCTION

The situation arises where one needs to consider the transient changes resulting from the departure of the reactor condition from the critical state. This arises under the the conditions of:

1. Startup,
2, Shutdown,
3. Accidental disturbances in the course of what is intended to be steady state operation.

The power produced during a reactor transient is one of the most important factors determining the degree of damage that can ensue from an accident.

The time dependent power production is related to the effective multiplication factor k_{eff} and the prompt and delayed neutron properties through the reactor kinetic equations. The spacial distribution of the reactor neutron flux is ignored in favor of an emphasis on its time behavior. The reactor is viewed as a point, hence the terminology of point reactor kinetics. In this regard, a distinction must be made between the behaviors of the prompt and delayed neutrons.

REACTIVITY DEFINITION

A critical reactor has an effective multiplication factor k_{eff} equal to unity. When a nuclear reactor deviates from criticality its effective multiplication factor can be larger or less than unity. In this case it has an "excess multiplication factor":

$$k_{ex} = k_{eff} - 1$$

which can be positive or negative.

The ratio of the excess multiplication factor to the effective multiplication factor is designated as the "excess reactivity" or simply, "reactivity":

$$\rho = \frac{k_{ex}}{k_{eff}} = \frac{k_{eff} - 1}{k_{eff}}$$

The reactivity describes the deviation of a reactor's effective multiplication factor from unity under time dependent conditions. For a steady-state or critical reactor the reactivity is zero.

If the deviation from criticality is small, such as caused by small deviations in temperature or voids during normal operation, the reactivity can be expressed as:

$$\rho \approx k_{ex} \approx k_{eff} - 1$$

In terms of the reactivity, the effective multiplication factor is expressed as:

$$k_{eff} = \frac{1}{1-\rho}$$

and the excess reactivity can be expressed as:

$$k_{ex} = \frac{\rho}{1-\rho}$$

The reactivity may be short lived because of some change in the system's temperature pressure or load. It may also develop over a long period of time because of fuel burnup, and the accumulation of fission products.

To hold the reactor power constant, means are devised to keep its reactivity constant such as the control rods or the chemical shim, which is a neutron absorber in the coolant or moderator.

PROMPT AND DELAYED NEUTRONS

About 99 percent of the fission neutron are designated as "prompt neutrons" since they are emitted within a short time interval 0f 10^{-17} sec of the fission process. The remaining neutrons are delayed in their emission in the process of the radioactive decay of the fission products to several minutes beyond the fission process itself and are designated as "delayed neutrons."

As an example, is the delayed neutron emission from the fission product isotope Br^{87} which has a half life of 55.6 seconds. The beta decay of Br^{87} through its two main branches of 2.6 and 8 MeV electrons leads to the formation of Kr^{87} in its ground state, and it subsequently decays through two successive beta emissions into the stable isotope Sr^{87}. Instead, it is possible for the delayed neutron precursor Br^{87} nucleus to beta decay into an excited state of the Kr^{87} nucleus at an energy of 5.5 MeV which is larger than the binding energy of a neutron in the Kr^{87} nucleus. In this case, the beta emission is followed by a neutron emission leading to the stable Kr^{86} isotope.

The fraction of delayed neutrons from U^{235} is only $\beta = 0.0065$, while it is smaller for Pu^{239} at $\beta = 0.0021$. The occurrence of delayed neutrons is crucial for the control of nuclear reactors. Their presence, even though small, provides a long time-constant that slows the dynamic time response of a nuclear reactor to make it controllable by the withdrawal and insertion of control rods containing nuclear absorbing materials such as boron.

The weighted average of the mean lifetime of the delayed neutrons is much larger than that of the prompt neutrons.

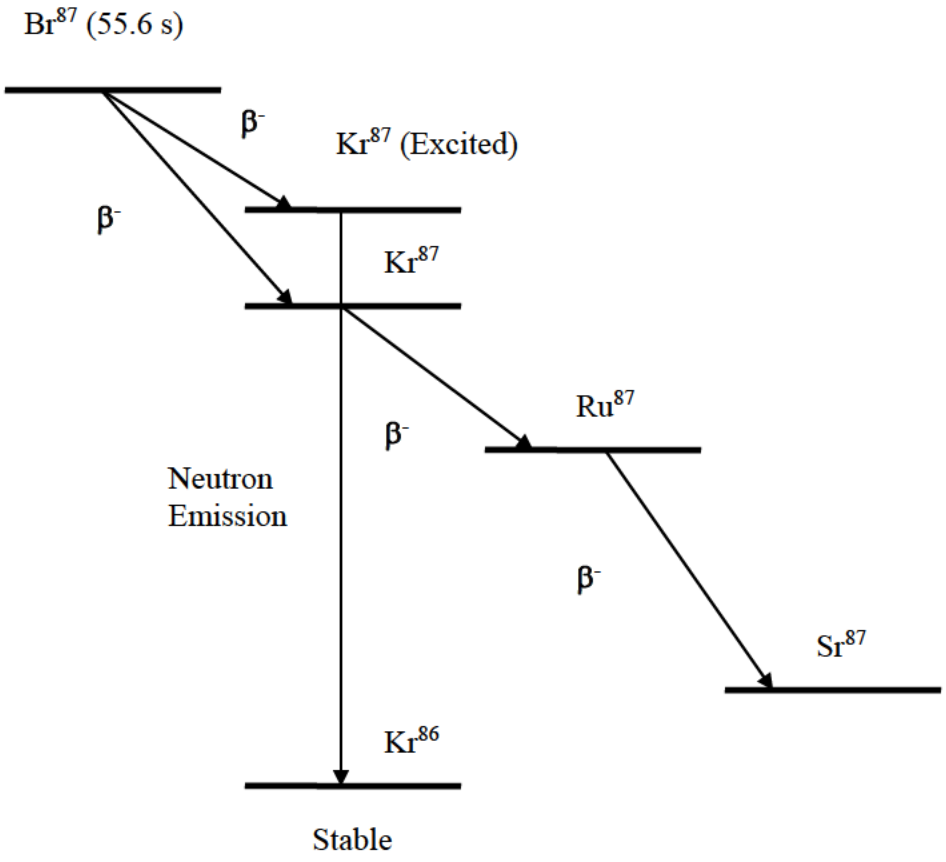

Figure 1. Delayed neutron emission from Bromine^{87}.

CLASSIFICATION OF REACTOR TIME DEPENDENT PROBLEMS

A classification according to the time scale on which appreciable changes in the neutron flux occur can be classified into the following classes.

Class I: Microseconds to milliseconds

These effects are due entirely to prompt fission neutrons. They occur so quickly that delayed neutrons and such effect as temperature changes have little opportunity to influence the time behavior of the neutron flux.

The initial stage of an ordinary reactor transient, accidents or the intentional case of an explosive device, are typical examples of this class.

Class II: Milliseconds to minutes

These include mostly ordinary controllable reactor transients such as reactor startup, adjustment or load changes.

The delayed neutrons play a major role in determining the time behavior. An example of delayed neutrons emission is that from Br^{87}.

Under these circumstances, the neutron flux equations are coupled to the heat transfer and coolant flow equations.

Class III: Minutes to days

These are due to unstable fission products of large absorption cross section such as xenon oscillations. Xenon production occurs through the fission of the fissile isotopes such as:

$$ {}_0n^1 + U^{235} \xrightarrow{\text{Fission}} Te^{135} \xrightarrow[11\text{sec}]{\beta^-} I^{135} \xrightarrow[6\,7hr]{\beta^-} Xe^{135} \xrightarrow[9\,16hr]{\beta^-} Cs^{135} \xrightarrow[2\,3x10^6\,yr]{\beta^-} Ba^{135} $$

Class IV: Days to years

These are due to change in the composition of the reactor core as the fuel is depleted and stable fission products, which are often poisons to the system, build up.

Class I events can be regarded as a special case of Class II, and Class III as a special case of Class IV. Classes I and II have a tine dependence arising from a lack of balance between the sources of neutrons and the leakage and absorption.

The time dependent diffusion equation states that the rate of change of the number of neutrons in a unit volume is just equal to the production rate of neutrons in the unit volume minus the loss rate:

$$ \frac{1}{\mathrm{v}} \frac{\partial \phi(\bar{r},t)}{\partial t} = S(\bar{r},t) + D\nabla^2\phi(\bar{r},t) - \Sigma_a\phi(\bar{r},t) \qquad (1) $$

DELAYED NEUTRON PARAMETERS

If we consider N delayed neutron groups, usually taken as 6, then the number of delayed neutrons produced per unit volume in the steady state is:

$$ n_d = \sum_{i=1}^{N} \lambda_i C_i(\bar{r}) \ \left[\frac{\text{neutrons}}{\text{cm}^3.\text{sec}}\right] $$

where: $C_i(\bar{r})$ is the concentration of the beta emitter i, which is a precursor of a delayed neutron emitter (2)

λ_i is a decay constant for the neutron emitter of the i-th type.

At steady state operation of a reactor, the generated fission products from fission are equal to those decaying through radioactive decay, thus:

$$ w_i \sum_f \phi(\bar{r}) - \lambda_i C_i(\bar{r}) = 0, \quad i = 1,2,3,\ldots,N \qquad (3) $$

where: w_i is the fraction of fissions which yield precursors of the i-th type.

The total number of delayed and prompt neutrons produced per unit volume per unit time will be:

$$\begin{aligned} n_t &= n_d + (1-\beta)\nu\textstyle\sum_f \varepsilon\phi(\bar{r}) \\ &= \sum_{i=1}^{N}\lambda_i C_i(\bar{r}) + (1-\beta)\nu\textstyle\sum_f \varepsilon\phi(\bar{r}) \end{aligned} \tag{4}$$

where: β is the delayed neutrons fraction from fissions.

This β is related to the fraction of neutrons from fission which is produced by the i-th delayed neutron precursor group by:

$$\beta = \sum_{i=1}^{N}\beta_i \tag{5}$$

POINT REACTOR KINETIC EQUATIONS FOR A NON STATIONARY ONE GROUP BARE REACTOR WITH DELAYED NEUTRONS

In this case the reactor is not critical and its effective multiplication factor will be different from unity:

$$\begin{aligned} k_{eff} &= k_\infty \ell_{th} \ell_f \\ &= \eta\varepsilon pf \frac{1}{(1+L^2B^2)}\frac{1}{(1+\tau B^2)} \\ &= \frac{\nu\textstyle\sum_f}{\textstyle\sum_{a\ fuel}}\varepsilon p \frac{\textstyle\sum_{a\ fuel}}{\textstyle\sum_a}\frac{1}{(1+L^2B^2)}\frac{1}{(1+\tau B^2)} \\ &= \frac{\nu\textstyle\sum_f}{\textstyle\sum_a}\varepsilon p \frac{1}{(1+L^2B^2)}\frac{1}{(1+\tau B^2)} \end{aligned} \tag{6}$$

where: $\sum_{a\ fuel}$ is the macroscopic fuel absorption cross section,

$\sum_a$ is the total absorption cross section,

B^2 is the geometric buckling.

The equations governing the precursors concentrations will be a modification of Eqn. 3 accounting for the time dependence as:

$$\frac{\partial C_i(\bar{r},t)}{\partial t} = w_i \textstyle\sum_f \phi(\bar{r},t) - \lambda_i C_i(\bar{r},t), \quad i = 1,2,3,\ldots,N. \tag{7}$$

The average energies of the delayed neutrons range from about 0.25 to 0.62 MeV. The balance equation for the thermal neutrons in terms of the flux is Eqn. 1, with a source term:

$$S(\bar{r},t) = (1-\beta)\nu\Sigma_f\,\varepsilon\phi(\bar{r},t)p_f + p\varepsilon_f\sum_{i=1}^{N}\lambda_i C_i(\bar{r},t)\,, \qquad (8)$$

Substituting Eqn. 8 into Eq. 1 yields:

$$\frac{1}{\mathrm{v}}\frac{\partial\phi(\bar{r},t)}{\partial t} = (1-\beta)\nu\Sigma_f\,\varepsilon\phi(\bar{r},t)p_f + p\varepsilon_f\sum_{i=1}^{N}\lambda_i C_i(\bar{r},t) + D\nabla^2\phi(\bar{r},t) - \Sigma_a\phi(\bar{r},t) \qquad (9)$$

It is reasonable to suppose that the spatial variation of the concentration of delayed neutrons precursors is proportional to that of the neutron flux and that this mode persists even though the magnitude of the flux changes with time. Thus let us assume:

$$\begin{aligned}\phi(\bar{r},t) &\equiv F(\bar{r}).\phi(t)\\ C_i(\bar{r},t) &\equiv F(\bar{r}).C_i(t)\end{aligned} \qquad (10)$$

where:

$$\nabla^2 F(\bar{r}) + B^2 F(\bar{r}) = 0, \qquad (11)$$

and the boundary condition at the extrapolated radius of the reactor:

$$F(R_{extrapolated}) = 0\,,$$

apply.

Substituting into Eq. 9 yields:

$$\frac{1}{\mathrm{v}}\frac{d\phi(t)}{dt} = (1-\beta)\nu\Sigma_f\,\varepsilon\phi(t)p_f + p\varepsilon_f\sum_{i=1}^{N}\lambda_i C_i(t) - (DB^2+\Sigma_a)\phi(t) \qquad (12)$$

$$\frac{dC_i(t)}{dt} = w_i\Sigma_f\,\phi(t) - \lambda_i C_i(t), \quad i = 1,2,3,\ldots,N. \qquad (13)$$

These reactor kinetics equations are coupled linear first order differential equations. The simple case when the coefficients are constant in time is known to have solutions which are exponentials.

SOLUTION OF THE REACTOR KINETIC EQUATIONS

For the case of constant coefficients, dividing into Σ_a, yields:

$$\frac{1}{\mathrm{v}\Sigma_a}\frac{d\phi(t)}{dt} = (1-\beta)\nu\frac{\Sigma_f}{\Sigma_a}\varepsilon\phi(t)p_{\ f} + \frac{p\varepsilon_{\ f}}{\Sigma_a}\sum_{i=1}^{N}\lambda_i C_i(t) - (\frac{D}{\Sigma_a}B^2 + 1)\phi(t)$$

The diffusion area is defined as:

$$L^2 = \frac{D}{\Sigma_a}$$

and dividing by $(1+L^2B^2)$, yields:

$$\frac{1}{\mathrm{v}\Sigma_a(1+L^2B^2)}\frac{d\phi(t)}{dt} = [(1-\beta)\nu\frac{\Sigma_f}{\Sigma_a}\varepsilon p_{\ f}\frac{1}{(1+L^2B^2)} - 1]\phi(t) + \frac{p\varepsilon_{\ f}}{\Sigma_a}\frac{1}{(1+L^2B^2)}\sum_{i=1}^{N}\lambda_i C_i(t)$$

Substituting for the effective multiplication factor:

$$k_{eff} = \frac{\nu\Sigma_f}{\Sigma_a}\varepsilon p\frac{1}{(1+L^2B^2)}\ell_f,$$

we get:

$$\frac{1}{\mathrm{v}\Sigma_a(1+L^2B^2)}\frac{d\phi(t)}{dt} = [(1-\beta)k_{eff} - 1]\phi(t) + k_{eff}\frac{1}{\nu\Sigma_f}\sum_{i=1}^{N}\lambda_i C_i(t)$$

We simplify the reactor kinetic equations by using the criticality equation and the definitions:

$$X_i(t) \equiv \frac{C_i(t)}{\nu\Sigma_f}, \qquad \beta_i \equiv \frac{w_i}{\nu} \tag{14}$$

Thus:

$$\overline{t}\frac{d\phi(t)}{dt} = [(1-\beta)k_{eff} - 1]\phi(t) + k_{eff}\sum_{i=1}^{N}\lambda_i X_i(t) \tag{15}$$

$$\frac{dX_i(t)}{dt} = \beta_i\phi(t) - \lambda_i X_i(t), \quad i = 1,2,3,\ldots,N. \tag{16}$$

where:

$$\bar{t} = \frac{1}{v\Sigma_a}\frac{1}{(1+L^2B^2)} = t_\infty \ell_{th} \tag{17}$$

is the average thermal neutron lifetime, and t_∞ is the infinite medium thermal neutron lifetime:

$$t_\infty = \frac{1}{v\Sigma_a} \tag{18}$$

We attempt the exponential solutions

$$\begin{aligned} \phi(t) &= \sum_{j=1}^{N'} a_j e^{\alpha_j t} \\ X_i(t) &= \sum_{j=1}^{N'} \gamma_{ij} e^{\alpha_j t},\ i = 1,2,3,\ \ldots\ , N \end{aligned} \tag{19}$$

Substitution into Eqn. 16 yields:

$$\sum_{j=1}^{N'} \gamma_{ij}\alpha_j e^{\alpha_j t} = \beta_i \sum_{j=1}^{N'} a_j e^{\alpha_j t} - \lambda_i \sum_{j=1}^{N'} \gamma_{ij} e^{\alpha_j t}, \quad i = 1,2,3,\ldots,N.$$

$$\sum_{j=1}^{N'} \gamma_{ij}(\alpha_j + \lambda_i) e^{\alpha_j t} = \beta_i \sum_{j=1}^{N'} a_j e^{\alpha_j t}$$

Since the exponential functions are linearly independent, we require the following condition for the existence of a solution:

$$\gamma_{ij}(\alpha_j + \lambda_i) = \beta_i a_j,\ i = 1,2,3,\ldots,N \tag{20}$$

Using Eqn. 20 and substituting Eqs. 19 into Eqn. 15, we get:

$$\bar{t}\sum_{j=1}^{N'} a_j\alpha_j e^{\alpha_j t} = [(1-\beta)k_{eff} - 1]\sum_{j=1}^{N'} a_j e^{\alpha_j t} + k_{eff}\sum_{i=1}^{N}\lambda_i \sum_{j=1}^{N'} \gamma_{ij} e^{\alpha_j t},\ i = 1,2,3,\ \ldots\ , N$$

$$\bar{t}\,\alpha_j \sum_{j=1}^{N'} a_j e^{\alpha_j t} = [(1-\beta)k_{eff} - 1]\sum_{j=1}^{N'} a_j e^{\alpha_j t} + k_{eff}\sum_{i=1}^{N}\lambda_i \frac{\beta_i}{(\alpha_j + \lambda_i)} \sum_{j=1}^{N'} a_j e^{\alpha_j t}$$

$$\bar{t}\,\alpha_j = [(1-\beta)k_{eff} - 1] + k_{eff}\sum_{i=1}^{N}\frac{\lambda_i\beta_i}{(\alpha_j + \lambda_i)},\ j = 1,2,3,\ \ldots\ , N'$$

Rearranging, we get:

$$[(1-\beta)k_{eff}-1]-\bar{t}\,\alpha_j=-k_{eff}\sum_{i=1}^{N}\frac{\lambda_i\beta_i}{(\alpha_j+\lambda_i)},\ j=1,2,3,\ \ldots\ ,N' \tag{21}$$

Thus Eqns. 20 and 21 are solutions to Eqns. 15 and 16, provided that the coefficients α_j are chosen to be the N' solutions of the characteristic Eq. 20 for any value of the α_j's.

REACTIVITY EQUATION OF REACTOR KINETICS

An expression for the effective multiplication factor can be written as a function of α from Eqn 21 as:

$$\begin{aligned}&[(1-\beta)k_{eff}]+k_{eff}\sum_{i=1}^{N}\frac{\lambda_i\beta_i}{(\alpha+\lambda_i)}=1+\bar{t}\,\alpha,\\ &k_{eff}=\frac{1+\bar{t}\,\alpha}{(1-\beta)+\sum_{i=1}^{N}\frac{\lambda_i\beta_i}{(\alpha+\lambda_i)}}\end{aligned} \tag{22}$$

In terms of the reactivity ρ:

$$\begin{aligned}\rho=\frac{\Delta k}{k_{eff}}=\frac{k_{eff}-1}{k_{eff}}&=\frac{\dfrac{1+\bar{t}\,\alpha}{(1-\beta)+\sum_{i=1}^{N}\frac{\lambda_i\beta_i}{(\alpha+\lambda_i)}}-1}{\dfrac{1+\bar{t}\,\alpha}{(1-\beta)+\sum_{i=1}^{N}\frac{\lambda_i\beta_i}{(\alpha+\lambda_i)}}}\\ &=\frac{1+\bar{t}\,\alpha-[(1-\beta)+\sum_{i=1}^{N}\frac{\lambda_i\beta_i}{(\alpha+\lambda_i)}]}{1+\bar{t}\,\alpha}\\ &=\frac{\bar{t}\,\alpha+\beta-\sum_{i=1}^{N}\frac{\lambda_i\beta_i}{(\alpha+\lambda_i)}]}{1+\bar{t}\,\alpha}\\ &=\frac{\bar{t}\,\alpha}{1+\bar{t}\,\alpha}+\frac{\sum_{i=1}^{N}\beta_i-\sum_{i=1}^{N}\frac{\lambda_i\beta_i}{(\alpha+\lambda_i)}]}{1+\bar{t}\,\alpha}\\ &=\frac{\bar{t}\,\alpha}{1+\bar{t}\,\alpha}+\frac{\sum_{i=1}^{N}\frac{\alpha\beta_i+\lambda_i\beta_i-\lambda_i\beta_i}{(\alpha+\lambda_i)}]}{1+\bar{t}\,\alpha}\end{aligned}$$

From which the reactivity can be expressed as equal to a continuous characteristic function $F(\alpha)$ as:

$$\rho = \frac{\bar{t}\,\alpha}{1+\bar{t}\,\alpha} + \frac{1}{1+\bar{t}\,\alpha}\sum_{i=1}^{N}\frac{\alpha\beta_i}{(\alpha+\lambda_i)}] = F(\alpha) \qquad (23)$$

DELAYED NEUTRON PARAMETERS

The time behavior of the delayed neutrons is empirically divided into a number of groups, typically six, characterized with a decay constant, a half-life, and a yield constant from the fission process.

Table 1. Six group delayed neutron parameters.

Group	Decay Constant λ_i [sec^{-1}]	Mean life $\tau_i = l/\lambda_i$ [sec]	Half life $\ln 2/\lambda_i$ [sec]	Fission yield w_i [n/fission]	Fractional yield β_i
Thermal fissions in U^{235}					
1	0.0124	80.645	55.72	0.00052	0.000215
2	0.0305	32.786	22.72	0.00346	0.001424
3	0.111	9.009	6.22	0.00310	0.001274
4	0.301	3.322	2.30	0.00624	0.002568
5	1.14	0.877	0.610	0.00182	0.000748
6	3.01	0.332	0.230	0.00066	0.000273
Total				**0.0158**	**0.0065**
Thermal fissions in U^{233}					
1	0.0126	79.365	55.00	0.00057	0.000224
2	0.0337	29.673	20.57	0.00197	0.000777
3	0.139	7.194	5.00	0.00166	0.000655
4	0.325	3.076	2.13	0.00184	0.000723
5	1.13	0.884	0.615	0.00034	0.000133
6	2.50	0.400	0.277	0.00022	0.000088
Total				**0.0066**	**0.0026**
Thermal fissions in Pu^{239}					
1	0.0128	78.125	54.28	0.00021	0.000073
2	0.0301	33.222	23.04	0.00182	0.000626
3	0.124	8.064	5.60	0.00129	0.000443
4	0.325	3.076	2.13	0.00199	0.000685
5	1.12	0.892	0.618	0.00052	0.000181
6	2.69	0.371	0.257	0.00027	0.000092
Total				**0.0061**	**0.0021**
Fast fissions in U^{238}					
1	0.0132	75.757	52.38	0.00054	0.000190
2	0.0321	31.152	21.58	0.00564	0.002030
3	0.139	7.194	5.00	0.00667	0.002400
4	0.358	2.793	1.93	0.01599	0.005740

5	1.41	0.709	0.490	0.00927	0.003330
6	4.02	0.248	0.172	0.00309	0.001110
Total				**0.0412**	**0.0148**

EFFECT OF EXISTENCE OF DELAYED NEUTRONS

Let the neutron population in a nuclear reactor at some t be n(t) and adopt a continuous population model. The effective multiplication factor can defined as the number of neutrons in a neutron generation after an average neutron lifetime relative to the number in the previous generation:

$$k_{eff} = \frac{n(t) + dn(t)}{n(t)} \tag{24}$$

The change in the neutron population can be written as:

$$dn(t) = n(t)k_{eff} - n(t) = n(t)(k_{eff} - 1) \tag{25}$$

The rate of change in the neutron population would be:

$$\frac{dn(t)}{dt} = n(t)\frac{(k_{eff} - 1)}{\tau} \tag{26}$$

where: τ is the average neutron lifetime between generations.

Separating the variables and using limits integration we get:

$$\int_{n_0}^{n(t)} \frac{dn(t)}{n(t)} = \frac{(k_{eff} - 1)}{\tau}\int_{0}^{t} dt$$

$$\ln\frac{n(t)}{n_0} = \frac{(k_{eff} - 1)}{\tau} \cdot t \tag{27}$$

$$n(t) = n_0 e^{\frac{(k_{eff} - 1)}{\tau} t}$$

The neutron flux and consequently the reactor power will rise with a period:

$$T = \frac{\tau}{k_{eff} - 1} \approx \frac{\tau}{\rho} \tag{28}$$

according to the exponential growth equation:

$$\phi(t) = \phi_0 e^{+\frac{t}{T}}$$
$$P(t) = P_0 e^{+\frac{t}{T}} \qquad (29)$$

Hypothetically, but not practically, the power in a reactor will rise on the basis of the prompt neutrons quite rapidly in the absence of rapid negative feedback or corrective action. For instance, for a graphite moderated reactor with k_{eff} = 1.001 or a 0.1 % excess reactivity, a period of 1 sec, the power increase in 5 seconds would be by a factor of:

$$\frac{P(t)}{P_0} = e^{+\frac{5}{1}} = 148.4$$

For an enriched uranium light water moderated reactor, the prompt neutron period would be hypothetically shorter around 0.1 sec resulting in an increase in power of:

$$\frac{P(t)}{P_0} = e^{+\frac{5}{0\,1}} = e^{+50} = 5.2\times10^{21}$$

Reactors do not just respond to prompt neutrons, since practically the delayed neutrons also affect the response of the reactor. The effect of the existence of delayed neutrons is an increase in the reactor period. The mean lifetime is increased from τ to the value:

$$(1-\beta)\tau + \sum_{i=1}^{6} \beta_i \tau_i \qquad (30)$$

For U^{235}, this is equal to 0.1 sec, with a resulting period of T = 0.1 / 0.001 = 100 sec and the power increase ratio within 5 seconds is a minor:

$$\frac{P(t)}{P_0} = e^{+\frac{5}{100}} = 1.05.$$

Practically then, he reactor response is so slower due to the presence of delayed neutrons, that enough time is available for corrective or negative feed back actions.

This provides a simple argument about the time behavior of a reactor. A more accurate representation is provided by the solution one group diffusion equation to the time dependent situation.

e-FOLDING TIME

The relative reactor power and flux can be written as:

$$\frac{P(t)}{P_0} = \frac{\phi(t)}{\phi_0} = e^{+\frac{t}{T}} = e^{+\frac{k_{eff}-1}{\tau}t}$$

The time at which the reactor power or flux changes by a factor of e = 2.718, is called the e -folding time. At this time:

$$\frac{P(t)}{P_0} = e = e^{+\frac{k_{eff}-1}{\tau}\tau_e} \Rightarrow \tau_e = \frac{\tau}{k_{eff}-1}$$

Thus we can also write:

$$\frac{P(t)}{P_0} = \frac{\phi(t)}{\phi_0} = e^{+\frac{t}{\tau_e}}$$

EXAMPLE

A hypothetical reactor without delayed neutrons with an average neutron lifetime τ of 10^{-3} sec subject to an accidental 0.1 percent step increase in the effective multiplication factor above criticality will have:

$$k_{ex} = k_{eff} - 1 = 1.001 - 1.0 = 0.001$$

$$\tau_e = \frac{\tau}{k_{eff}-1} = \frac{0.001}{0.001} = 1 \text{ sec}$$

Its power and flux ratios will be:

$$\frac{P(t)}{P_0} = \frac{\phi(t)}{\phi_0} = e^{+\frac{t}{\tau_e}} = e^{+\frac{t}{1}} = e^{t}$$

For this reactor, the flux and power will increase at the rate of e or 2.718 times per second.

A realistic reactor accounting for the delayed neutrons with a period τ of 50 sec will have:

$$\tau_e = \frac{\tau}{k_{eff}-1} = \frac{50}{0.001} = 5\times10^{4} \text{ sec}$$

$$\frac{P(t)}{P_0} = \frac{\phi(t)}{\phi_0} = e^{+\frac{t}{\tau_e}} = e^{+\frac{t}{5\times10^{4}}} = e^{2\times10^{-5}t}$$

For this reactor, the flux and power will increase at the minor rate of just 1.00002 times per second.

INHOUR UNIT OF REACTIVITY

A common unit used to measure the reactivity is the inverse-hour; abbreviated as inhour. One inhour is the amount of reactivity that would make the period of a reactor equal to 1 hour or 1x60x60 = 3,600 seconds. Thus:

$$T = \frac{\tau}{k_{eff} - 1}$$

$$= \frac{k_{eff}}{k_{eff} - 1} \frac{\tau}{k_{eff}}$$

$$= \frac{\tau}{\rho k_{eff}} = 3,600$$

$$\rho = \frac{\tau}{3,600\ k_{eff}}$$

EXAMPLE

In the previous example, a 1 hour period would require the reactivity to be:

$$\rho = \frac{50}{3,600 \times 1.001} = 0.0139$$

DOLLAR UNIT OF REACTIVITY

The reactivity can be expressed in terms of the total delayed neutron fraction:

$$\beta = \sum_{i=1}^{N} \beta_i \, .$$

This introduces a unit commonly used to measure the reactivity.

The dollar ($) unit is defined as the amount of reactivity equal to the delayed neutron fraction for the particular fuel in the reactor:

$$1\ \$ = \frac{\rho}{\beta}$$

$$1\ cent = \frac{1}{100}\ \$ = \frac{\rho}{100\beta}$$

The cent unit is one hundredth of the dollar unit. This practice is convenient in that a reactivity value in dollars will essentially produce the same rate of flux and power rise for reactors containing the fissile fuels U^{233}, U^{235} and Pu^{239}.

SOLUTION OF THE ONE GROUP TIME DEPENDENT DIFFUSION REACTIVITY EQUATION

Knowing the neutron lifetime and the delayed neutron properties, one can plot the characteristic function $F(\alpha)$ as a function of α and determine the discrete values α_i where the value of the reactivity intersects with the characteristic function.

The function $F(\alpha)$ has the following properties:

$$F(0) = 0,$$

$$\lim_{\alpha \to \pm\infty} F(\alpha) = 1,$$

$$\lim_{\varepsilon \to 0} F(-\lambda_i \pm \varepsilon) = \mp\infty,$$

$$\lim_{\varepsilon \to 0} F(-\frac{1}{t} \pm \varepsilon) = \mp\infty,$$

For a given positive or negative ρ there are N' = N + 1 solutions for α. All solutions are real.

When ρ is positive, there are six negative solutions and one positive one.

When ρ is negative, there are seven negative solutions.

In a specific problem, the general solutions in Eqns. 19 must satisfy some initial conditions of the system, such as:

$$\phi(t=0) = \phi_0, X_i(t=0) = X_{i0}, i = 1, 2, 3, \ldots, N.$$

The previous conditions provide a sufficient number of relations from which to compute the coefficients a_j and γ_{ij} since the α_j values are known.

Thus:

$$\phi_0 = \sum_{j=1}^{N+1} a_j \ , \ \gamma_{ij}(\alpha_j + \lambda_i) = a_j \beta_i$$

with:

$$\sum_{j=1}^{N+1} \gamma_{ij} = X_{i0} \ , \ i = 1, 2, 3, \ldots, N \ , \ j = 1, 2, 3, \ldots, N, N+1,$$

which results in $(N+1)^2$ equations for $(N+1)^2$ unknowns.

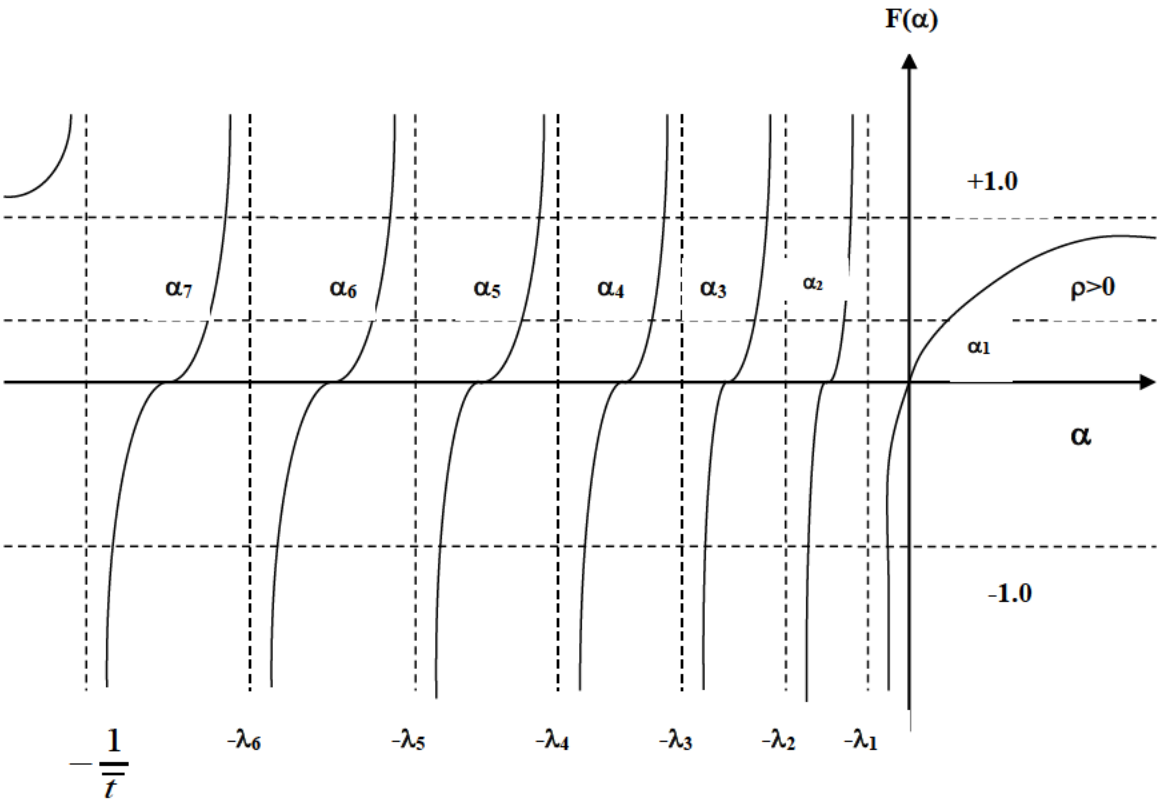

Figure 2. Characteristic function F(α) for 6 groups of delayed neutrons.

For instance for N=6, the unknowns are:

$$\begin{array}{ccccccc} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 \\ \gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} & \gamma_{15} & \gamma_{16} & \gamma_{17} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \gamma_{24} & \gamma_{25} & \gamma_{26} & \gamma_{27} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} & \gamma_{35} & \gamma_{36} & \gamma_{37} \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & \gamma_{44} & \gamma_{45} & \gamma_{46} & \gamma_{47} \\ \gamma_{51} & \gamma_{52} & \gamma_{53} & \gamma_{54} & \gamma_{55} & \gamma_{56} & \gamma_{57} \\ \gamma_{61} & \gamma_{62} & \gamma_{63} & \gamma_{64} & \gamma_{65} & \gamma_{66} & \gamma_{67} \end{array}$$

ASYMPTOTIC BEHAVIOR

The time behavior of the neutron flux is quite complicated in the immediate time after the reactivity addition. After a sufficient time, the terms involving:

$$e^{\alpha_j t} \; for \; j > 1,$$

will be negligible in comparison with the first term $e^{\alpha_1 t}$. Thus after some time:

$$\begin{aligned} \phi(t) &\approx e^{\alpha_1 t} \\ X_i(t) &\approx e^{\alpha_1 t} \end{aligned} \tag{31}$$

For a small and positive reactivity, let us expand in a Taylor's series about $\alpha = 0$, thus:

$$\rho \approx F(\alpha = 0) + \frac{dF(\alpha = 0)}{d\alpha}\alpha_1 + \ldots$$

and :

$$\alpha_1 \approx \frac{\rho - F(\alpha = 0)}{\frac{dF(\alpha = 0)}{d\alpha}} = \frac{\rho}{\frac{dF(\alpha = 0)}{d\alpha}}$$

From which:

$$\alpha_1 \approx \frac{\rho}{\bar{t} + \sum_{i=1}^{N} \frac{\beta_i}{\lambda_i}} \tag{32}$$

Thus the neutron flux becomes:

$$\phi(t) \approx e^{+\frac{\rho}{\bar{t} + \sum_{i=1}^{N} \frac{\beta_i}{\lambda_i}} t} \tag{33}$$

The reactor period becomes:

$$T \approx \left(\bar{t} + \sum_{i=1}^{N} \frac{\beta_i}{\lambda_i} \right) . \frac{1}{\rho} \tag{34}$$

Comparison to the previously obtained result for a reactor without a delayed neutrons contribution:

$$T' = \frac{t^*}{\rho},$$

shows that the effect of delayed neutrons is to increase the effective mean life-time of the neutrons.

The same result can be obtained heuristically as:

$$t^* \approx (1-\beta)\bar{t} + \beta_1(\bar{t} + \frac{1}{\lambda_1}) + \beta_2(\bar{t} + \frac{1}{\lambda_2}) + ... + \beta_N(\bar{t} + \frac{1}{\lambda_N})$$
$$\approx \bar{t} + \sum_{i=1}^{N} \frac{\beta_i}{\lambda_i} \tag{35}$$

where the first term accounts for the prompt neutrons, and the other terms account for the delayed neutron groups.

The stable reactor period is defined as:

$$T = \frac{1}{\alpha_1} \tag{36}$$

PROMPT CRITICALITY STATE

The prompt multiplication factor for prompt neutrons can be written as:

$$(1-\beta)k_{eff} = 1,$$
$$k_{eff} = \frac{1}{1-\beta}$$

The prompt criticality condition can be written as:

$$\rho = \frac{k_{eff} - 1}{k_{eff}} = \frac{\frac{1}{1-\beta} - 1}{\frac{1}{1-\beta}} = \beta$$

From the perspective of reactor safety, very small periods must be avoided. If the reactor period is too small, the reactor power may increase at a rate faster than the shutdown systems rate of intervention.

The reactor period decreases with increased reactivity. For small positive reactivity additions, the period is long and nearly independent of the neutron lifetime. There exists a precipitous drop in the period length for short average neutron generation time about the prompt critical value of:

$$\rho = \beta = 0.0065 \text{ for } U^{235}.$$

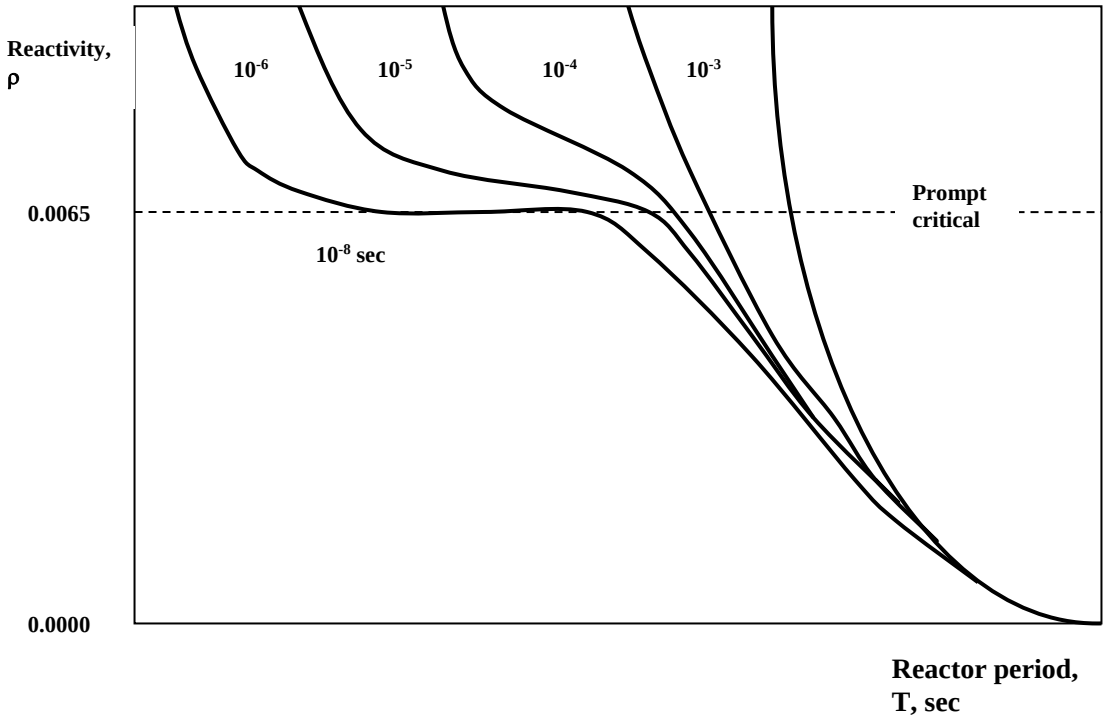

Figure 3. Reactor reactivity and period for different neutron generation times.

ONE DELAYED NEUTRONS GROUP POINT REACTOR KINETIC MODEL ANALYTICAL SOLUTION

This result can be obtained by setting N=1, leading to:

$$\frac{\beta}{\lambda} = \sum_{i=1}^{N} \frac{\beta_i}{\lambda_i}$$

Thus we can write:

$$\phi(t) = a_1 e^{\alpha_1 t} + a_2 e^{\alpha_2 t}$$
$$X(t) = \gamma_1 e^{\alpha_1 t} + \gamma_2 e^{\alpha_2 t}$$

If we denote the initial conditions at time t = 0 as:

$$\phi_0, X_0$$

Thus:

$$\phi_0 = a_1 + a_2$$
$$X_0 = \gamma_1 + \gamma_2$$

and from the equations:

$$\gamma_{ij}(\alpha_j + \lambda_i) = a_j \beta_i \ , \ j = 1,2$$

we get:

$$\gamma_1(\alpha_1 + \lambda) = a_1 \beta$$
$$\gamma_2(\alpha_2 + \lambda) = a_2 \beta$$

We have four equations in four unknowns:

$$a_1, a_2, \gamma_1, \gamma_2,$$

which can be solved to obtain their values.

A relation between ϕ_0 and X_0 can be obtained at time t = 0 for a single delayed neutron group:

$$\lambda C_0 = \nu\beta \Sigma_F \phi_0$$
$$\lambda X_0 = \beta\phi_0$$

Thus:

$$X_0 = \gamma_1 + \gamma_2$$
$$\frac{\beta}{\lambda}\phi_0 = \gamma_1 + \gamma_2$$

Substituting for γ_1 and γ_2 we get:

$$\frac{\beta}{\lambda}\phi_0 = \frac{a_1\beta}{\alpha_1 + \lambda} + \frac{a_2\beta}{\alpha_2 + \lambda}$$
$$\frac{1}{\lambda}\phi_0 = \frac{a_1}{\alpha_1 + \lambda} + \frac{a_2}{\alpha_2 + \lambda}$$

Substituting for a_2:

$$\frac{1}{\lambda}\phi_0 = \frac{a_1}{\alpha_1 + \lambda} + \frac{(\phi_0 - a_1)}{\alpha_2 + \lambda}$$

From which:

$$a_1\left(\frac{1}{\alpha_1+\lambda}-\frac{1}{\alpha_2+\lambda}\right)=\phi_0\left(\frac{1}{\lambda}-\frac{1}{\alpha_2+\lambda}\right)$$

$$a_1=\frac{\alpha_2(\alpha_1+\lambda)}{\lambda(\alpha_2-\alpha_1)}\phi_0$$

Similarly:

$$a_2=\phi_0-a_1=-\frac{\alpha_1(\alpha_2+\lambda)}{\lambda(\alpha_2-\alpha_1)}\phi_0$$

The reactivity equation becomes:

$$\rho=\frac{\bar{t}\,\alpha}{1+\bar{t}\,\alpha}+\frac{1}{1+\bar{t}\,\alpha}\frac{\alpha\beta}{\alpha+\lambda}$$

$$\rho=\frac{\alpha}{1+\bar{t}\,\alpha}\left(\bar{t}+\frac{\beta}{\alpha+\lambda}\right)$$

This is a quadratic equation in α:

$$\bar{t}\,(\rho-1)\alpha^2+[\rho(1+\bar{t}\,\lambda)-(\beta+\bar{t}\,\lambda)]\alpha+\lambda\rho=0$$

Using the solution to the quadratic equation:

$$ax^2+bx+c=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

the quadratic has the solutions for a positive root α_1 and a negative root α_2 as:

$$\bar{t}\,(\rho-1)\alpha^2+[\rho(1+\bar{t}\,\lambda)-(\beta+\bar{t}\,\lambda)]\alpha+\lambda\rho=0$$

$$\alpha_{1,2}=-\frac{[\rho(1+\bar{t}\,\lambda)-(\beta+\bar{t}\,\lambda)]}{2\bar{t}\,(\rho-1)}\pm\frac{1}{2}\sqrt{\left(\frac{[\rho(1+\bar{t}\,\lambda)-(\beta+\bar{t}\,\lambda)]}{\bar{t}\,(\rho-1)}\right)^2-\frac{4\lambda\rho}{\bar{t}\,(\rho-1)}}$$

FLUX TRANSIENT DUE TO A POSITIVE STEP REACTIVITY INSERTION

A typical transient in a nuclear reactor may involve the following numerical values of the kinetic parameters:

$$\beta = 0.00755$$
$$\lambda = 0.076 \text{ [sec]}^{-1}$$
$$\rho = 0.001$$
$$\bar{t} = 0.001 \text{ [sec]}$$

From these values we can calculate:

$$\alpha_1 = 0.01145 \text{ [sec]}^{-1}$$
$$\alpha_2 = -\ 6.644 \text{ [sec]}^{-1}$$

The flux coefficients become:

$$a_1 = 1.148682\phi_0$$
$$a_2 = -0.148682\phi_0$$

The expression for the flux becomes:

$$\phi(t) = \phi_0(1.1486821e^{0\ 01145t} - 0.11486821e^{-6\ 644t})$$

The second exponential decays rapidly and essentially 0.5 sec after introducing the reactivity, the neutron flux rises with the stable reactor period:

$$\frac{\phi(t)}{\phi_0} \cong 1.148682e^{0\ 01145\ t}$$

To display such a transient, his was incorporated into the following computational procedure.

```
!       One delayed neutrons group point reactor kinetic model
!       Calculation of flux ratio in a reactor transient
!       Procedure saves output to file:output1
!       This output file can be exported to a plotting routine
!       M. Ragheb, University of Illinois
        program flux_transient
        integer :: steps=100
        real flux_ratio(101),t(101),beta,rho,lambda,taverage,deltat
!       Calculational time step
        deltat=0.1
!       Delayed neutron fraction
        beta=0.00755
!       Delayed group decay constant
        lambda=0.076
!       Reactivity insertion
        rho=0.001
!       Average thermal neutron lifetime
        taverage=0.001
```

```
      write(*,*) beta,lambda,rho,taverage
!     Open output file
      open(10,file='output1')
!     Calculate roots of reactivity equation
      x1=taverage*(rho-1.0)
      x2=(rho*(1.0+taverage*lambda))-(beta+(taverage*lambda))
      x3=x2/x1
      x4=x3*x3-(4.0*lambda*rho/x1)
      x5=(sqrt(x4))/2.0
      x6=x2/(2.0*x1)
      alpha1=-x6+x5
      alpha2=-x6-x5
      write(*,*) alpha1, alpha2
!     Calculate flux coefficients
      x7=alpha2-alpha1
      a1=alpha2*(alpha1+lambda)/(lambda*x7)
      a2=-alpha1*(alpha2+lambda)/(lambda*x7)
      write(*,*)a1,a2
!     Calculate flux ratio
      steps = steps + 1
      do i =  1, steps
            t(i)=(i-1)*deltat
            flux_ratio(i)=a1*exp(alpha1*t(i))+a2*exp(alpha2*t(i))
!     Display results on screen
            write(*,*) t(i), flux_ratio(i)
!           pause
      end do
!  Write final result
      do i =  1, steps
!     Write results on output file
      write(10,*) t(i), flux_ratio(i)
      end do
      end
```

Figure 4. Procedure for a reactor transient involving the addition of a reactivity $\rho = 0.001$ using the one group of delayed neutrons point reactor kinetic model.

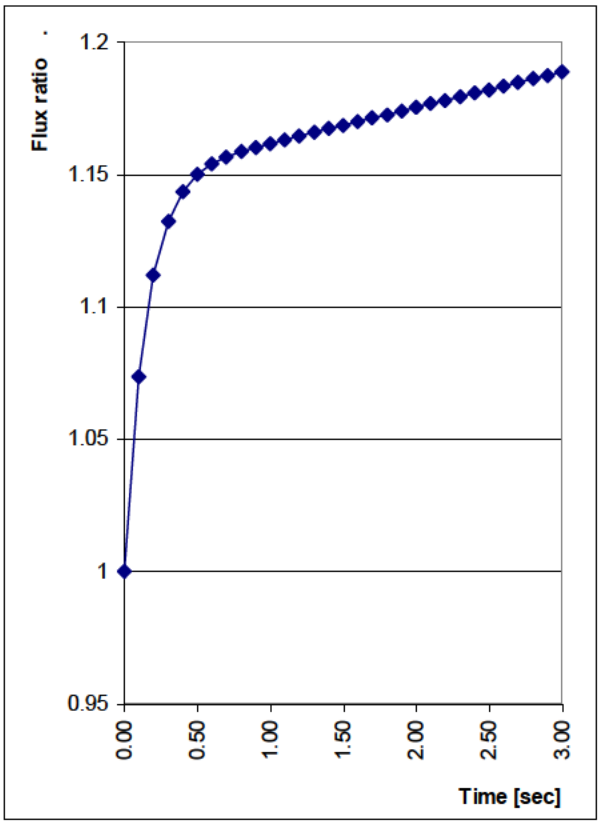

Figure 5. During the first few seconds the flux ratio increases and at about half a second it rises according to the stable reactor period.

RAMP REACTIVITY ADDITION

The step reactivity previously discussed has a certain level of mathematical idealization since for a reactivity change to be instantaneous it would have to occur over a time period shorter than the prompt neutron generation time. The treatment of the step reactivity is acceptable for small reactivity additions since in this case the reactor period is long.

For large reactivity additions in postulated safety analyses, the time behavior of the reactivity addition should be accounted for. For instance, in the case of a ramp reactivity addition, we write:

$$\rho(t) = ct$$

Such a ramp reactivity addition in safety analyses can be associated with several postulated causes such as the uncontrollable withdrawal of a simple, or a bank of control rods, the voiding of the liquid metal in a liquid metal cooled reactor, or the compaction of a highly enriched core.

For safety analysis considerations, the amount of postulated core damage in a reactor transient would be closely correlated to the reactivity addition rate:

$$\frac{d\rho(t)}{dt} = c\,,$$

rather than the total reactivity available.

Several reasons can be advanced for the argument. First, the rate of reactivity addition determines the time available to detect the transient and shut down the reactor before a prompt critical condition is reached. Second, if a prompt criticality condition were reached, the power rise would be so rapid that the electro-mechanical shutdown systems would not act fast enough to terminate the excursion. Third, either inherent feedback mechanisms or core damage would terminate the prompt critical stage before all the reactivity is added.

The time at which the reactor would reach prompt criticality would be:

$$t_{prompt} = \frac{\beta}{c}\,,$$

and the power ratio:

$$\frac{P(t_{prompt})}{P_0}$$

would increase with the increased ramp rate c.

To prevent a potentially prompt critical excursion, either the transient must be detected at a time sufficiently long before t_{prompt}, or the reactor design must depend on the inherent feedback mechanisms in such a way as to assure that they override the reactivity addition mechanism before prompt criticality is reached.

REACTOR STARTUP

The inherent feedback mechanisms tend to be negligible if the reactor power is not at least a few percent of the full power value such as under startup or zero power testing conditions.

Only well-informed and educated reactor operators are aware that a reactor could reach a prompt criticality stage at a power level below that at which any of the negative temperature feedback mechanisms would become functional. At this stage, none of the instrumentation measuring the reactor thermal parameters such as the core outlet temperature or primary system pressure would detect the initiation of the transient before prompt criticality is reached.

In this case, a reactor period meter would be depended upon to detect the transient initiation regardless of the rector power level. Considering the time dependence of the reactor power ratio:

$$\frac{P(t)}{P_0} = e^{+\frac{t}{T}}$$

The rate of power increase would be:

$$\frac{dP(t)}{dt} = P_0 e^{\frac{t}{T}} \frac{1}{T} = P(t)\frac{1}{T}$$

From which we get a quantity that can be measured instantaneously:

$$T(t) = \frac{P(t)}{\frac{dP(t)}{dt}}$$

Without proper precautions the startup of a new reactor can be a hazardous situation. Precautions should include the incremental loading of the fuel into the core with the control rods fully inserted and then withdrawn to check on the reactivity of the fuel configuration. Also a neutron flux should be maintained through the presence of an external source. For a fixed source, the inverse count rate from several detectors gives an indication of how rapidly criticality is approached relative to the weight of the fuel or the number of fuel assemblies loaded into the core.

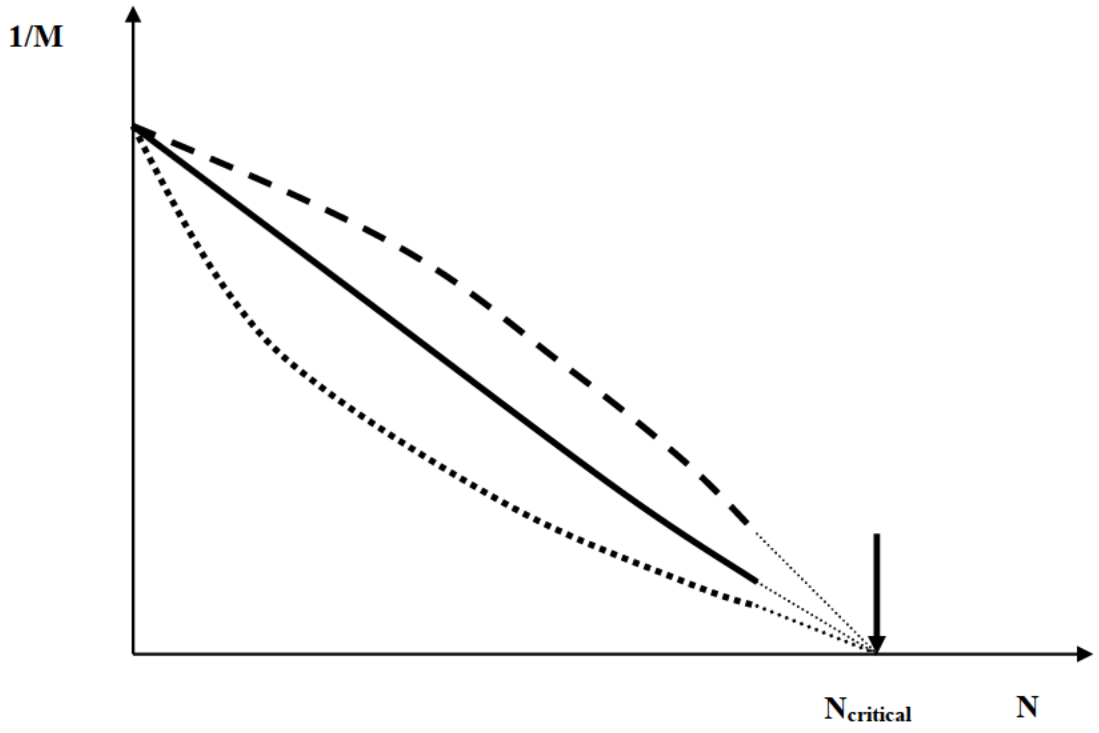

Figure 6. Reciprocal counting rate from several detectors is used with an external source to determine the critical number of fuel assemblies during reactor startup.

The startup transient safety analysis id hypothesized as consisting of an addition to a ramp reactivity to the reactivity state of the subcritical system as:

$$\rho(t) = -\rho_0 + ct$$

As the ramp reactivity addition is increased, the power at which the reactor reaches prompt criticality becomes smaller. For this reason it is desirable during startup to have a small rate of reactivity addition through small control rod worths and small control rods withdrawal speeds.

In addition since P_0 is proportional to the external source strength and it is desired to have as small a power ratio as possible, then as large a source as possible is needed.

SHUTDOWN REACTIVITY

The effectiveness of a nuclear reactor shutdown depends on several factors:

1. The speed at which the control rods can be brought into the core.
2. The rods incremental worth, defined as the decrease in reactivity per unit length of rod insertion.
3. The fraction of a second time needed to transmit the electrical signals from the neutron flux detectors to the control rods actuators.
4. The several seconds of time required to drive the rods far enough into the core to turn it into a subcritical state.

It must be noted that the incremental rod worth is largest when the rod tip passes through the high flux region at the core midplane. About 0.7 second is needed to attain a full control rod worth. For this reason the safe design of nuclear reactors requires the dependence on the feedback mechanisms to provide a faster action.

FEEDBACK REACTIVITY COEFFICIENTS

Changes with time in the operating parameters of the nuclear reactor such as temperature, lead to reactivity changes. These are described by different coefficients of reactivity. Some of these coefficients are negative leading to a decrease in the reactor's power, and some are positive. The safe operation of a nuclear reactor requires that their overall sum be negative leading to an overall negative feedback effect damping any perturbations in the reactor power level.

The effects of the coefficients of reactivity are felt rapidly. They are different from other long term reactivity changes which are slow and long ranging such as the one due to the change in the composition of the fuel due to fuel burnup, or fissile fuel production in a breeder reactor. Fuel depletion causes a decrease in the power level, whereas breeding causes an increase. The accumulation of the fission products absorbs neutrons and decreases the value of fuel utilization factor and hence the power level. The temporary buildup of Xe^{135} after reactor shutdown affects the ability to restart the reactor.

TEMPERATURE COEFFICIENT OF REACTIVITY

The temperature coefficient of reactivity for a temperature T is defined as:

$$\alpha_T = \frac{d\rho}{dT}$$

It may be negative or positive, leading to a decrease or increase in the reactivity following a decrease or increase in temperature. A reactor with a negative temperature coefficient of reactivity is inherently safe.

The temperature changes to a reactor affect the reactivity in three ways:

1. Altering the mean energy of the thermal neutrons in a thermal reactor, whereas the fast neutrons are unaffected trough the nuclear temperature coefficient.
2. Affecting the densities of the reactor components such the coolant, fuel and moderator through the density temperature coefficient.
3. Changing the dimensions and volume of the reactor's core through the volume temperature coefficient.

Summing the three neutrons (n), density (d) and volumetric (v) contributions:

$$\alpha_T = \alpha_n + \alpha_d + \alpha_v$$
$$\frac{d\rho}{dT} = \frac{\partial\rho_n}{\partial T} + \frac{\partial\rho_d}{\partial T} + \frac{\partial\rho_v}{\partial T}$$

The temperature coefficient is usually negative. It is largest in homogeneous reactors where the density component predominates, resulting in the relative inherent safety of such systems. Heterogeneous reactors that are solid fuelled are appreciably affected by density changes only if moderated by a liquid, such as in the case of the Pressurized Water Reactor (PWR) concept and solid moderated and circulating liquid-fuelled reactors in general. Heterogeneous reactors in which the fuel and moderators are in solid form such as the graphite moderated gas cooled and sodium cooled reactors, are not substantially affected by the density changes and their temperature coefficients are relatively small.

We can write for the temperature coefficient of reactivity:

$$\alpha_T = \frac{d\rho}{dT} = \frac{d}{dT}\frac{k_{eff}-1}{k_{eff}} = \frac{d}{dT}(1-\frac{1}{k_{eff}}) = \frac{1}{k_{eff}^2}\frac{dk_{eff}}{dT} \approx \frac{1}{k_{eff}}\frac{dk_{eff}}{dT}$$

Since:

$$k_{eff} = k_\infty \ell_f \ell_{th}$$

k_∞ is the infinite medium neutron multiplication factor,

where : ℓ_f is the fast neutrons non-leakage probability,

ℓ_{th} is the thermal neutrons non-leakage probability.

$$\alpha_T = \frac{1}{k_\infty}\frac{\partial k_\infty}{\partial T} + \frac{1}{\ell_f}\frac{\partial \ell_f}{\partial T} + \frac{1}{\ell_{th}}\frac{\partial \ell_{th}}{\partial T}$$

Using the four-factor formula:

$$k_\infty = \eta \varepsilon p f \ ,$$

the first term can be expressed as:

$$\alpha_{k_\infty} = \frac{1}{k_\infty}\frac{d(\eta\varepsilon pf)}{dT} = \frac{1}{\eta}\frac{\partial \eta}{\partial T} + \frac{1}{\varepsilon}\frac{\partial \varepsilon}{\partial T} + \frac{1}{p}\frac{\partial p}{\partial T} + \frac{1}{f}\frac{\partial f}{\partial T}$$

The second term is negligible since the fast fissions are not affected by changes in the thermal neutrons spectrum.

The third term is not important in a homogeneous reactor, but becomes important in a heterogeneous reactor.

The regeneration factor η term is temperature dependent since it involves the ratio of two cross section that are a function of the neutron energy:

$$\eta = \nu \frac{\sigma_f}{\sigma_a}$$

Since an increase in the temperature of the moderator leads to a corresponding increase in the mean thermal energy of the moderator molecules and a shift to a higher peak in the Maxwellian neutron energy distribution, the ratio of the fission to absorption cross section as affected leading to an appreciable temperature coefficient of the regeneration factor. In natural uranium, added to the contribution from U^{235}, there will also exist a contribution from the variation of the ratio of the radiative captures in U^{235} to U^{238} as a function of temperature.

The temperature coefficient for the fuel utilization factor:

$$f = \frac{\Sigma_{a\ fuel}}{\Sigma_{a\ fuel} + \Sigma_{a\ \text{moderator}} + \Sigma_{a\ structure} + \Sigma_{a\ control} + \ldots}$$

will be negligible if the cross sections possess the same 1/v behavior such as in the case of a natural uranium or U^{235} core. If Pu^{239} is present, a strong positive temperature coefficient of f takes place as a result in the shift of the peak of the thermal spectrum towards the region of a large Pu^{239} resonance at 0.3 eV.

An important effect occurs in water moderated reactors where an increase in temperature decreases the water density causing an increase in the fuel utilization factor f, hence introduces a positive reactivity.

The temperature coefficient of the resonance escape probability p is significant in heterogeneous reactors. In a water cooled reactor, a higher temperature causes some water expulsion from the core resulting in a lower moderator to fuel ratio. The resonance escape probability has an exponential dependence on it, so that an increase in temperature reduces the value of p resulting in a negative temperature coefficient.

Light water reactors are undermoderated whereas the addition of hydrogen nuclei produces a positive effect on the resonance escape probability which is larger than the negative effect on the thermal utilization factor. The overall effect of moderator expansion is negative. In addition, the lower moderator density enhances leakage of both the thermal and fast neutron components.

If the moderator has a nuclear shim in the form of a burnable poison such a s boric acid to control the reactivity, the increase in the value of f could be significant enough to lead to a net positive effect.

The thermal neutrons nonleakage probability:

$$\ell_{th} = \frac{1}{1 + L^2 B^2}, \quad B^2 = \left(\frac{\pi}{R}\right)^2, \text{ for a spherical core}$$

includes the buckling that varies inversely to the size of the reactor, as well as the diffusion length L. The increase in the size of the reactor due to thermal expansion decreases the buckling, and enhances the leakage leading to an increase in the reactivity. Some fast reactors are given a pancake shape to encourage high neutron leakage.

DOPPLER COEFFICIENT OF REACTIVITY

This effect primarily affects the resonance escape probability temperature coefficient in addition to the effect in the change in the moderator to fuel ratio. The target nucleus in neutron interactions is not totally at rest since it is in thermal equilibrium and hence possesses a certain level of vibrational energy. Its magnitude is small relative to the neutron energy and can be ignored in most cases. The thermal motion is random in nature and has a component along the same direction as the incident interacting neutron. This spreads the energy of the neutron around its actual energy. This leads to a broadening of the resonances in the cross sections resonance region and is similar to the sound and light Doppler effect; hence its name.

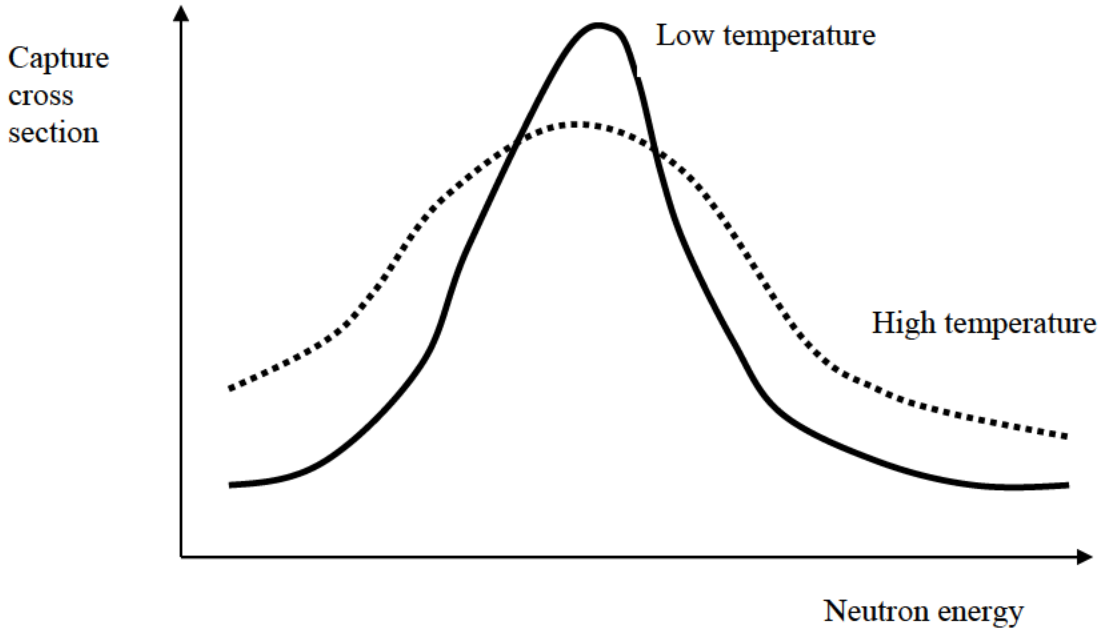

Figure 4. The nuclear Doppler Effect involves a widening of the neutron absorption cross sections resonances with an increase in temperature.

Doppler broadening of the resonances also lowers their peaks even though the area under them remains constant. In principle, there should be no change in the reactivity if the nuclei were well dispersed and dilute such as in a homogeneous reactor where the U^{238} atoms are essentially isolated from one another. In most situations of heterogeneous reactors, however, a strong self-shielding effect occurs which causes a saturation in the cross sections inside the fuel.

For a lumped fuel element most of the U^{238} is shielded from the flux of resonance neutrons because the peak cross section in a resonance is so high that they are absorbed near the surface of a fuel rod. The reduction in the peak height caused by the Doppler broadening reduces the flux depression in the fuel resulting in less self screening when the fuel is hot, and causing an increased absorption of neutrons in the U^{238} resonances.

The Doppler effect thus results in an increased rate of resonance absorption since it broadens the resonance without affecting its already saturated height. There will therefore be a Doppler coefficient of reactivity which could be positive for a fissile fuel such as U^{235}, or could be negative due to a fissionable fuel such as U^{238}, which primarily absorbs neutrons without fissioning.

In low-enrichment reactors, the effect of the presence of U^{238} predominates, the Doppler effect is negative though its magnitude is small. Its main importance is that its effect is prompt since the fission energy shows up directly in the fuel material causing an immediate increase in their thermal vibrations. This does not occur substantially in the coolant or moderator whose heat transport mechanism accounting for their temperature rise is slower.

In the case of fast reactors, two situations arise. The neutron spectum could be hard with a high average neutron energy and a narrow distribution, or it could be soft with a lower average energy and a wide distribution.

Soft spectrum conditions occur if some reactor components such as sodium have a moderating effect on the neutrons, or if a moderator such as BeO is intentionally added to the core. Since a soft spectrum spans a larger part of the resonance region than a hard one, this will result in a negative Doppler coefficient.

On the other hand, a hard spectrum reactor with highly enriched fuel such as in Naval Propulsion reactors, could have a positive Doppler coefficient. The design of the reactor must preclude this situation from occurring.

NUCLEAR TEMPERATURE COEFFICIENT

This arises because of the effect of temperature on the thermal diffusion length L:

$$L^2 = \frac{D}{\Sigma_a} \approx \frac{1}{3\Sigma_s \Sigma_a}$$

The thermal diffusion length L varies with temperature due to the variation of the macroscopic absorption cross section with it. Since the microscopic absorption cross section follows a 1/v behavior with neutron velocity and consequently energy, the diffusion length L will increase with an increase in temperature.

The magnitude of the nuclear temperature coefficient is proportional to L^2, and is usually negative.

In sodium-cooled reactors, the expansion of the sodium with temperature reduces its small moderating ability and increases the average neutron energy. This increases the regeneration factor and consequently the infinite medium multiplication factor, resulting in a positive nuclear coefficient.

DENSITY TEMPERATURE COEFFICIENT

This arises because of the change of the density of core materials with increasing temperature. An increase in temperature decreases the density and hence the macroscopic scattering and absorption cross sections;

$$\Sigma = \frac{1}{\lambda} = N'\sigma = \frac{\rho}{M} A_v \sigma$$

This increases the mean free paths of the neutrons and therefore neutron leakage from the surface of the core. The density temperature coefficient is proportional to an average linear coefficient of thermal expansion of the core materials and is usually negative.

A heterogeneous light water moderated reactor with a large moderator to fuel ratio, or over moderated reactors, may have a positive density coefficient. Because light water is a neutron absorber, an initial decrease in its density would actually increase its multiplication factor since the decrease in neutron moderation has a larger effect than the decreased moderation. Further decrease in density, though reverses the effect leading to a negative density coefficient.

VOLUME TEMPERATURE COEFFICIENT

This is caused by an overall increase of the reactor size and hence affects primarily the geometric buckling. For a spherical core, the geometrical buckling is expressed in terms of the core radius R as:

$$B_g^2 = \left(\frac{\pi}{R}\right)^2$$

It is proportional to an average linear coefficient of expansion of core materials and core vessel. It is usually positive but numerically is the smallest in magnitude of the three components of the temperature coefficient.

VOID COEFFICIENT OF REACTIVITY

In the situation of an inadvertent power surge, boiling could occur in a liquid moderator after it reaches its saturation temperature, forming vapor bubbles or voids. The void coefficient is defined as:

$$\alpha_\alpha = \frac{d\rho}{d\alpha}$$

where α here is the void fraction or the fraction of the volume of the moderator that is in the form of vapor.

Boiling Water Reactors (BWRs) are designed to operate with a fraction of the moderator in vapor form. Pressurized Water Reactors (PWRs) can operate with a power surge of up to 20 percent increase in the nominal power level leading to temporary boiling. In a reactor with a normal fuel to moderator ratio, an increase in the void fraction results in decreased moderation and consequently a negative void coefficient.

In an overmoderated reactor with a high moderator to fuel ratio, the void coefficient may become positive. In sodium cooled reactors, sodium voiding results in a positive void coefficient.

In heavy water moderated reactors the degree of moderation needed for operation with natural uranium fuel leads to a small positive void coefficient. The subdivision in the pipework in the pressure tube design allows a loss of coolant transient to be easily terminated by the normal reactivity control mechanism. The largest positive void coefficient occurs in the heavy water moderated CANDU designs, and the boiling light water designs such as the Chernobyl RBMK-1000. Larger diameter fuel elements are used in this case to increase the time constant of heat transfer from the fuel to the coolant allowing the control system to reduce the reactor power before a significant void is formed.

The void coefficient is important in sodium cooled reactors. Sodium voiding produces two important effects. A reduction in scattering increases the average neutron energy in the core increasing the value of the regeneration factor coupled to an increase in fast fissions in U^{238} which has a threshold fission cross section at about 6.5 MeV. This leads to a positive reactivity contribution. The decrease in the coolant density, on the other hand leads to increased leakage which cancels the first effect. The void coefficient can be

made always negative by a suitable choice of the geometry of the core and by increasing the negative value of the Doppler broadening coefficient.

PRESSURE COEFFICIENT OF REACTIVITY

This is due to changes in the reactor pressure and is defined as:

$$\alpha_p = \frac{d\rho}{dp}$$

Since liquid densities are insensitive to pressure changes, moderators that remain in a solid of liquid form are unaffected by the pressure changes.

A BWR, however, is extremely sensitive to the pressure changes and has a large pressure coefficient which is usually positive. These pressure changes can occur as a result of varying steam demand from the turbine. If the turbine governor is closed, the reactor pressure rises causing the voids to collapse; thus increasing moderation and hence the reactor power level.

A large pressure coefficient occurs at low system pressures in BWRs and causes the reactor to be unstable. It decreases rapidly with increasing pressure and becomes relatively small at high system pressures above several hundred psia.

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